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## REFERENCES:

- ❖ *Mechanical Engineering Design by Shigley, 8th Edition, 2008.*
- ❖ *Mechanical Engineering Design by Shigley, 9th Edition, 2011.*
- ❖ *Machine Design by Khurmi, Fourteenth Edition, 2005.*

# CHAPTER ONE

## Load and Stress Analysis

### *1.1 Definition:*

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

### *1.2 Classifications of Machine Design:*

The machine design may be classified as follows :

1. **Adaptive design.** In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.
2. **Development design.** This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.
3. **New design.** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows :

- (a) **Rational design.** This type of design depends upon mathematical formulae of principle of mechanics.
- (b) **Empirical design.** This type of design depends upon empirical formulae based on the practice and past experience.

(c) **Industrial design.** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) **Optimum design.** It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(e) **System design.** It is the design of any complex mechanical system like a motor car.

(f) **Element design.** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) **Computer aided design.** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

### ***1.3 Design Considerations:***

Sometimes the strength required of an element in a system is an important factor in the determination of the geometry and the dimensions of the element. In such a situation we say that strength is an important design consideration. When we use the expression design consideration, we are referring to some characteristic that influences the design of the element or, perhaps, the entire system. Usually quite a number of such characteristics must be considered and prioritized in a given design situation. Many of the important ones are as follows (not necessarily in order of importance):

- |                                    |                                       |
|------------------------------------|---------------------------------------|
| 1. Functionality                   | 14. Noise                             |
| 2. Strength/stress                 | 15. Styling                           |
| 3. Distortion/deflection/stiffness | 16. Shape                             |
| 4. Wear                            | 17. Size                              |
| 5. Corrosion                       | 18. Control                           |
| 6. Safety                          | 19. Thermal properties                |
| 7. Reliability                     | 20. Surface                           |
| 8. Manufacturability               | 21. Lubrication                       |
| 9. Utility                         | 22. Marketability                     |
| 10. Cost                           | 23. Maintenance                       |
| 11. Friction                       | 24. Volume                            |
| 12. Weight                         | 25. Liability                         |
| 13. Life                           | 26. Remanufacturing/resource recovery |

Some of these characteristics have to do directly with the dimensions, the material, the processing, and the joining of the elements of the system. Several characteristics may be interrelated, which affects the configuration of the total system.

### **1.4 Standards and Codes:**

A standard is a set of specifications for parts, materials, or processes intended to achieve uniformity, efficiency, and a specified quality. One of the important purposes of a standard is to place a limit on the number of items in the specifications so as to provide a reasonable inventory of tooling, sizes, shapes, and varieties.

A code is a set of specifications for the analysis, design, manufacture, and construction of something. The purpose of a code is to achieve a specified degree of safety, efficiency, and performance or quality. It is important to observe that safety codes do not imply absolute safety. In fact, absolute safety is impossible to obtain. See the book, pg. 12.

### **1.5 Stress and Strength**

The survival of many products depends on how the designer adjusts the maximum stresses in a component to be less than the component's strength at critical locations. The designer must allow the maximum stress to be less than the strength by a sufficient margin so that despite the uncertainties, failure is rare.

In focusing on the stress-strength comparison at a critical (controlling) location, we often look for "strength in the geometry and condition of use." Strengths are the magnitudes of stresses at which something of interest occurs, such as the proportional limit, 0.2 percent-offset yielding, or fracture. In many cases, such events represent the stress level at which loss of function occurs.

Strength is a property of a material or of a mechanical element. The strength of an element depends on the choice, the treatment, and the processing of the material. Consider, for example, a shipment of springs. We can associate strength with a specific spring. When this spring is incorporated into a machine, external forces are applied that result in load-induced stresses in the spring, the magnitudes of which depend on its geometry and are independent of the material and its processing. If the spring is removed from the machine undamaged, the stress due to the external forces will return to zero. But the strength remains as one of the properties of the spring. Remember, then, that strength is an inherent property of a part, a property built into the part because of the use of a particular material and process.

### **1.6 Uncertainty**

Uncertainties in machinery design abound. Examples of uncertainties concerning stress and strength include

- Composition of material and the effect of variation on properties.
- Variations in properties from place to place within a bar of stock.
- Effect of processing locally, or nearby, on properties.

- Effect of nearby assemblies such as weldments and shrink fits on stress conditions.
- Effect of thermomechanical treatment on properties.
- Intensity and distribution of loading.
- Validity of mathematical models used to represent reality.
- Intensity of stress concentrations.
- Influence of time on strength and geometry.
- Effect of corrosion.
- Effect of wear.
- Uncertainty as to the length of any list of uncertainties.

Engineers must accommodate uncertainty. Uncertainty always accompanies change. Material properties, load variability, fabrication fidelity, and validity of mathematical models are among concerns to designers.

There are mathematical methods to address uncertainties. The primary techniques are the deterministic and stochastic methods. The deterministic method establishes a design factor based on the absolute uncertainties of a loss-of-function parameter and a maximum allowable parameter. Here the parameter can be load, stress, deflection, etc. Thus, the design factor  $n_d$  is defined as

$$n_d = \frac{\text{loss-of-function parameter}}{\text{maximum allowable parameter}} \quad 1-1$$

### 1.7 Design Factor and Factor of Safety

A general approach to the allowable load versus loss-of-function load problem is the deterministic design factor method, and sometimes called the classical method of design. The fundamental equation is Eq. (1-1) where  $n_d$  is called the design factor. All loss-of-function modes must be analyzed, and the mode leading to the smallest design factor governs. After the design is completed, the actual design factor may change as a result of changes such as rounding up to a standard size for a cross section or using off-the-shelf components with higher ratings instead of employing what is calculated by using the design factor. The factor is then referred to as the factor of safety,  $n$ . The factor of safety has the same definition as the design factor, but it generally differs numerically.

Since stress may not vary linearly with load, using load as the loss-of-function parameter may not be acceptable. It is more common than to express the design factor in terms of a stress and a relevant strength. Thus Eq. (1-1) can be rewritten as

$$n_d = \frac{\text{loss-of-function strength}}{\text{allowable stress}} = \frac{S}{\sigma(\text{or } \tau)}$$

1-2

The stress and strength terms must be of the same type and units. Also, the stress and strength must apply to the same critical location in the part.

**Factor of Safety** is defined, in general, as the ratio of the maximum stress to the working stress. Mathematically,

$$\text{Factor of safety} = \text{Maximum stress} / \text{Working or design stress}$$

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \text{Yield point stress} / \text{Working or design stress}$$

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\text{Factor of safety} = \text{Ultimate stress} / \text{Working or design stress}$$

This relation may also be used for ductile materials.

**Note:** The above relations for factor of safety are for static loading.

### 1.8 Load

It is defined as any external force acting upon a machine part. The following four types of the load are important from the subject point of view:

1. **Dead or steady load.** A load is said to be a dead or steady load, when it does not change in magnitude or direction.
2. **Live or variable load.** A load is said to be a live or variable load, when it changes continually.
3. **Suddenly applied or shock loads.** A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.
4. **Impact load.** A load is said to be an impact load, when it is applied with some initial velocity.

**Note:** A machine part resists a dead load more easily than a live load and a live load more easily than a shock load.

### 1.9 Stress

When some external system of forces or loads acts on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as **unit stress** or simply a **stress**. It is denoted by a Greek letter sigma ( $\sigma$ ). Mathematically,

$$\text{Stress, } \sigma = P/A$$

Where

$P = \text{Force or load acting on a body, and}$

$A = \text{Cross-sectional area of the body.}$

In S.I. units, the stress is usually expressed in Pascal (Pa) such that  $1 \text{ Pa} = 1 \text{ N/m}^2$ . In actual practice, we use bigger units of stress i.e. mega Pascal (MPa) and gig Pascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

And

$$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

### 1.10 Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as **unit strain** or simply a **strain**. It is denoted by a Greek letter epsilon ( $\epsilon$ ). Mathematically,

$$\text{Strain, } \epsilon = \delta l / l \text{ or } \delta l = \epsilon.l$$

where

$\delta l = \text{Change in length of the body, and}$

$l = \text{Original length of the body.}$

### 1.11 Tensile Stress and Strain

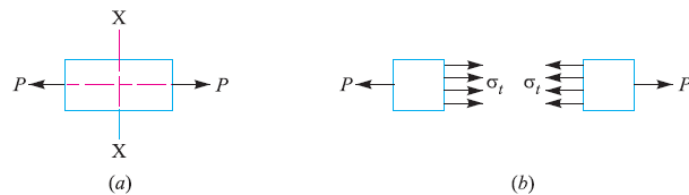


Fig. 1.1. Tensile stress and strain.

When a body is subjected to two equal and opposite axial pulls  $P$  (also called tensile load) as shown in Fig. 1.1 (a), then the stress induced at any section of the body is known as tensile stress as shown in Fig. 1.1 (b). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as **tensile strain**.

Let  $P =$  Axial tensile force acting on the body,  
 $A =$  Cross-sectional area of the body,  
 $l =$  Original length, and  
 $\delta l =$  Increase in length.

$\therefore$  Tensile stress,  $\sigma_t = P/A$

and tensile strain,  $\varepsilon_t = \delta l / l$

### 1.12 Compressive Stress and Strain:

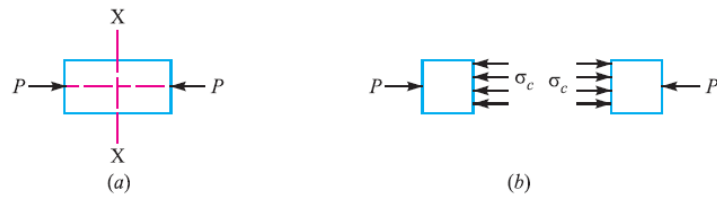


Fig.1.2. Compressive stress and strain

When a body is subjected to two equal and opposite axial pushes  $P$  (also called compressive load) as shown in Fig. 1.2 (a), then the stress induced at any section of the body is known as **compressive stress** as shown in Fig. 1.2 (b). A little consideration will show that due to the compressive load, there will be an increase in cross-sectional area and a decrease in length of the body. The ratio of the decrease in length to the original length is known as **compressive strain**.

Let  $P =$  Axial compressive force acting on the body,  
 $A =$  Cross-sectional area of the body,  
 $l =$  Original length, and  
 $\delta l =$  Decrease in length.

Compressive stress,  $\sigma_c = P/A$

and compressive strain,  $\varepsilon_c = \delta l / l$

**Note :** In case of tension or compression, the area involved is at right angles to the external force applied.



### 1.13 Young's Modulus or Modulus of Elasticity:

Hooke's law states that when a material is loaded within elastic limit, the stress is directly proportional to strain, i.e.

$$\sigma \propto \epsilon \quad \text{or} \quad \sigma = E \cdot \epsilon$$

$\therefore$

$$E = \frac{\sigma}{\epsilon} = \frac{P \times l}{A \times \delta l}$$

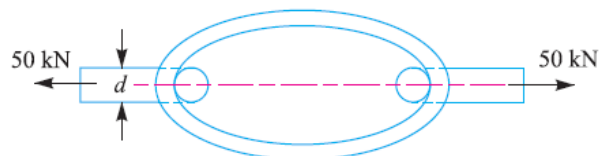
where E is a constant of proportionality known as Young's modulus or modulus of elasticity. In S.I. units, it is usually expressed in GPa i.e. GN/m<sup>2</sup> or kN/mm<sup>2</sup>. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (E) for the materials commonly used in engineering practice.

**Table 1.1. Values of E for the commonly used engineering materials**

<u>Material</u>	<u>Modulus of elasticity (E) in GPa i.e. GN/m<sup>2</sup> or kN/mm<sup>2</sup></u>
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

**Example:** A coil chain of a crane required to carry a maximum load of 50 kN, is shown in Figure below. Find the diameter of the link stock, if the permissible tensile stress in the link material is not to exceed 75 MPa.



**Solution.** Given :  $P = 50 \text{ kN} = 50 \times 10^3 \text{ N}$  ;  $\sigma_t = 75 \text{ MPa} = 75 \text{ N/mm}^2$

Let  $d = \text{Diameter of the link stock in mm.}$

$$\text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854 d^2$$

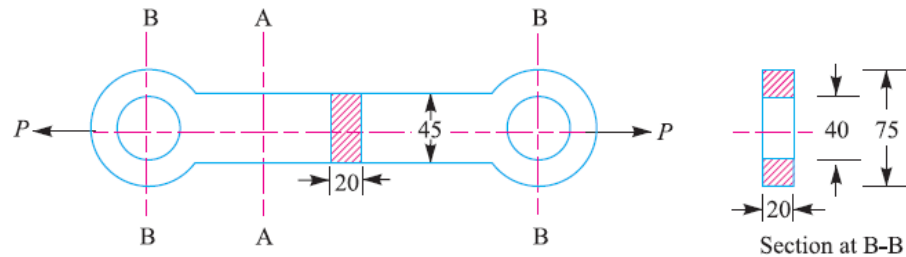
We know that the maximum load (P),

$$50 \times 10^3 = \sigma_t \cdot A = 75 \times 0.7854 d^2 = 58.9 d^2$$

$$d^2 = 50 \times 10^3 / 58.9 = 850 \text{ or } d = 29.13 \text{ say } 30 \text{ mm}$$

Ans.

**Example:** A cast iron link, as shown in Figure below is required to transmit a steady tensile load of 45 kN. Find the tensile stress induced in the link material at sections A-A and B-B. All dimensions in mm.



**Solution.** Given:  $P = 45 \text{ kN} = 45 \times 10^3 \text{ N}$

#### Tensile stress induced at section A-A

We know that the cross-sectional area of link at section A-A,

$$A_1 = 45 \times 20 = 900 \text{ mm}^2$$

Tensile stress induced at section A-A,

$$\sigma_{t1} = \frac{P}{A_1} = \frac{45 \times 10^3}{900} = 50 \text{ N/mm}^2 = 50 \text{ MPa} \quad \text{Ans.}$$

#### Tensile stress induced at section B-B

We know that the cross-sectional area of link at section B-B,

$$A_2 = 20(75 - 40) = 700 \text{ mm}^2$$

Tensile stress induced at section B-B,

$$\sigma_{t1} = \frac{P}{A_1} = \frac{45 \times 10^3}{700} = 64.3 \text{ N/mm}^2 = 64.3 \text{ MPa} \quad \text{Ans.}$$

**Example:** A hydraulic press exerts a total load of 3.5 MN. This load is carried by two steel rods, supporting the upper head of the press. If the safe stress is 85 MPa and  $E = 210 \text{ kN/mm}^2$ , find :

1. diameter of the rods, and
2. extension in each rod in a length of 2.5 m.

**Solution.** Given :  $P = 3.5 \text{ MN} = 3.5 \times 10^6 \text{ N}$  ;  $\sigma_t = 85 \text{ MPa} = 85 \text{ N/mm}^2$  ;  $E = 210 \text{ kN/mm}^2 = 210 \times 10^3 \text{ N/mm}^2$  ;  $l = 2.5 \text{ m} = 2.5 \times 10^3 \text{ mm}$

#### 1. Diameter of the rods

Let

$d = \text{Diameter of the rods in mm.}$

$$\text{Area,} \quad A = \frac{\pi}{4} \times d^2 = 0.7854 d^2$$

Since the load  $P$  is carried by two rods, therefore load carried by each rod,

$$P_1 = P/2 = 3.5 \times 10^6 / 2 = 1.75 \times 10^6 \text{ N}$$

We know that load carried by each rod ( $P_1$ ),  $1.75 \times 10^6 = \sigma_t \cdot A = 85 \times 0.7854 d^2 = 66.76 d^2$

$$1.75 \times 10^6 / 66.76 = 26213 \text{ or } d = 162 \text{ mm} \quad \text{Ans.}$$

## 2. Extension in each rod

Let  $\delta l = \text{Extension in each rod.}$

We know that Young's modulus ( $E$ ),

$$210 \times 10^3 = P_1 \times l / A \times \delta l = \sigma_t \times l / \delta l = 85 \times 2.5 \times 10^3 / \delta l = 212.5 \times 10^3 / \delta l$$

$$\delta l = 212.5 \times 10^3 / (210 \times 10^3) = 1.012 \text{ mm} \quad \text{Ans.}$$

### 1.14 Shear Stress and Strain:

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called **shear stress**.

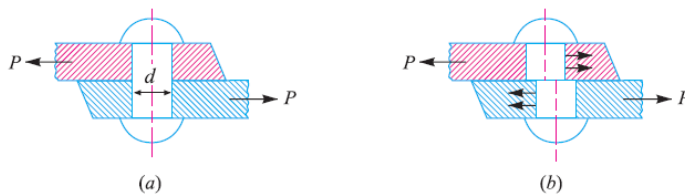


Fig. Single shearing of a riveted joint.

The corresponding strain is known as shear strain and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau ( $\tau$ ) and phi ( $\phi$ ) respectively. Mathematically,

$$\text{Shear stress, } \tau = \text{Tangential force} / \text{Resisting area}$$

Consider a body consisting of two plates connected by a rivet as shown in Fig. (a). In this case, the tangential force  $P$  tends to shear off the rivet at one cross-section as shown in Fig. (b). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in **single shear**. In such a case, the area

resisting the shear off the rivet,  $A = \frac{\pi}{4} d^2$

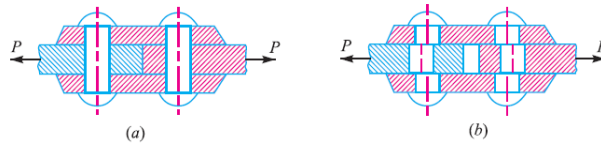
and shear stress on the rivet cross-section,  $\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d^2} = \frac{4P}{\pi d^2}$

Now let us consider two plates connected by the two cover plates as shown in Fig. (a). In this case, the tangential force  $P$  tends to shear off the rivet at two cross-sections as shown in Fig. (b). It may be noted that when the tangential force is resisted by two cross-sections of the rivet (or when the shearing takes place at two cross-sections of the rivet), then the rivets are said to be in **double shear**. In such a

case, the area resisting the shear off the rivet,  $A = 2 \times \frac{\pi}{4} d^2$  ... (For double shear)

and shear stress on the rivet cross-section, 
$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} d^2} = \frac{2P}{\pi d^2}$$

Fig. Double shearing of a riveted joint.



**Notes :** 1. All lap joints and single cover butt joints are in single shear, while the butt joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter ‘d’ is to be punched in a metal plate of thickness ‘t’, then the area to be sheared,

$$A = \pi d \times t$$

and the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi d \times t \times \tau_u$$

where

$\tau_u$  = Ultimate shear strength of the material of the plate.

### 1.15 Shear Modulus or Modulus of Rigidity:

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \cdot \phi \quad \text{or} \quad \tau / \phi = C$$

where  $\tau$  = Shear stress,

$\phi$  = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

The following table shows the values of modulus of rigidity (C) for the materials in everyday use:

**Table 1.2. Values of C for the commonly used materials.**

<u>Material</u>	<u>Modulus of rigidity (C) in GPa i.e. GN/m<sup>2</sup> or kN/mm<sup>2</sup></u>
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

**Example:** Calculate the force required to punch a circular blank of 60 mm diameter in a plate of 5 mm thick. The ultimate shear stress of the plate is 350 N/mm<sup>2</sup>.

**Solution.** Given:  $d = 60 \text{ mm}$  ;  $t = 5 \text{ mm}$  ;  $\tau_u = 350 \text{ N/mm}^2$

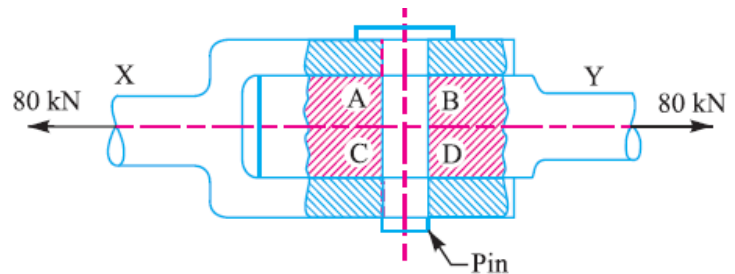
We know that area under shear,

$$A = \pi d \times t = \pi \times 60 \times 5 = 942.6 \text{ mm}^2$$

and force required to punch a hole,

$$P = A \times \tau_u = 942.6 \times 350 = 329\,910 \text{ N} = 329.91 \text{ kN} \quad \text{Ans.}$$

**Example:** A pull of 80 kN is transmitted from a bar X to the bar Y through a pin as shown in Fig. If the maximum permissible tensile stress in the bars is 100 N/mm<sup>2</sup> and the permissible shear stress in the pin is 80 N/mm<sup>2</sup>, find the diameter of bars and of the pin.



**Solution.** Given :  $P = 80 \text{ kN} = 80 \times 10^3 \text{ N}$ ;  $\sigma_t = 100 \text{ N/mm}^2$  ;  $\tau = 80 \text{ N/mm}^2$

### Diameter of the bars

Let  $D_b =$  Diameter of the bars in mm.

$$\text{Area, } A_b = \pi/4 (D_b)^2 = 0.7854(D_b)^2$$

We know that permissible tensile stress in the bar ( $\sigma_t$ ),

$$100 = P/A_b = 80 \times 10^3 / 0.7854(D_b)^2 = 101\,846 / (D_b)^2$$

$$(D_b)^2 = 101\,846 / 100 = 1018.46$$

Or  $D_b = 32 \text{ mm} \quad \text{Ans.}$

### Diameter of the pin

Let  $D_p =$  Diameter of the pin in mm.

Since the tensile load  $P$  tends to shear off the pin at two sections i.e. at AB and CD, therefore the pin is in double shear.

Resisting area,

$$A_p = 2 \times (\pi/4 (D_p)^2) = 1.571 (D_p)^2$$

We know that permissible shear stress in the pin ( $\tau$ ),

$$80 = P/A_p = 80 \times 10^3 / 1.571 (D_p)^2 = 50.9 \times 10^3 / (D_p)^2$$

$$(D_p)^2 = 50.9 \times 10^3 / 80 = 636.5 \quad \text{or} \quad D_p = 25.2 \text{ mm} \quad \text{Ans.}$$

### 1.16 Bearing Stress:

A localized compressive stress at the surface of contact between two members of a machine part, that are relatively at rest is known as **bearing stress** or **crushing stress**. The bearing stress is taken into account in the design of riveted joints, cotter joints, knuckle joints, etc. Let us consider a riveted joint subjected to a load  $P$  as shown in Fig. In such a case, the bearing stress or crushing stress (stress at the surface of contact between the rivet and a plate),

$$\sigma_b \text{ (or } \sigma_c) = P/d \cdot t \cdot n$$

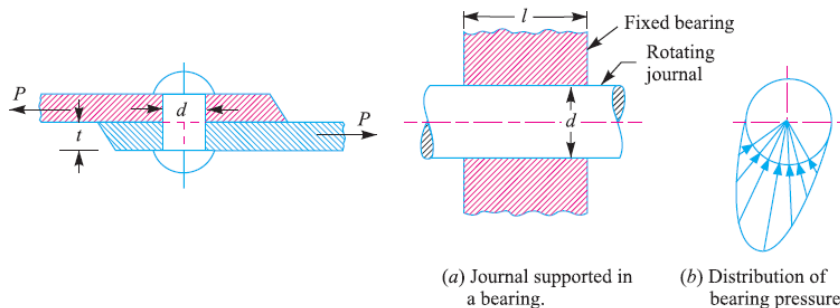
where

$d$  = Diameter of the rivet,

$t$  = Thickness of the plate,

$d \cdot t$  = Projected area of the rivet, and

$n$  = Number of rivets per pitch length in bearing or crushing.



It may be noted that the local compression which exists at the surface of contact between two members of a machine part that are in relative motion, is called bearing pressure (not the bearing stress). This term is commonly used in the design of a journal supported in a bearing, pins for levers, crank pins, clutch lining, etc. Let us consider a journal rotating in a fixed bearing as shown in Fig. (a). The journal exerts a bearing pressure on the curved surfaces of the brasses immediately below it.

The distribution of this bearing pressure will not be uniform, but it will be in accordance with the shape of the surfaces in contact and deformation characteristics of the two materials. The distribution of bearing pressure will be similar to that as shown in Fig. (b). Since the actual bearing pressure is difficult to determine, therefore the average bearing pressure is usually calculated by dividing the load to the projected area of the curved surfaces in contact. Thus, the average bearing pressure for a journal supported in a bearing is given by

$$p_b = P/l \cdot d$$

where

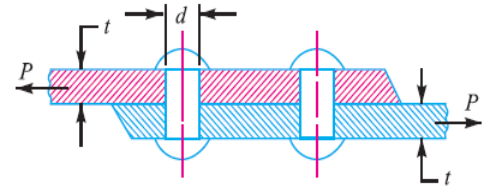
$p_b$  = Average bearing pressure,

$P$  = Radial load on the journal,

$l$  = Length of the journal in contact, and

$d$  = Diameter of the journal.

**Example:** Two plates 16 mm thick are joined by a double riveted lap joint as shown in Fig. The rivets are 25 mm in diameter. Find the crushing stress induced between the plates and the rivet, if the maximum tensile load on the joint is 48 kN.



**Solution.** Given :  $t = 16 \text{ mm}$  ;  $d = 25 \text{ mm}$  ;  $P = 48 \text{ kN} = 48 \times 10^3 \text{ N}$

Since the joint is double riveted, therefore, strength of two rivets in bearing (or crushing) is taken.

We know that crushing stress induced between the plates and the rivets,

$$\sigma_c = P / d \cdot t \cdot n = 48 \times 10^3 / 25 \times 16 \times 2 = 60 \text{ N/mm}^2 \quad \text{Ans.}$$

**Example:** A journal 25 mm in diameter supported in sliding bearings has a maximum end reaction of 2500 N. Assuming an allowable bearing pressure of  $5 \text{ N/mm}^2$ , find the length of the sliding bearing.

**Solution.** Given :  $d = 25 \text{ mm}$  ;  $P = 2500 \text{ N}$  ;  $p_b = 5 \text{ N/mm}^2$

Let  $l =$  Length of the sliding bearing in mm.

We know that the projected area of the bearing,

$$A = l \times d = l \times 25 = 25l \text{ mm}^2$$

Bearing pressure ( $p_b$ ),

$$5 = p/A = 2500/25l = 100/l \quad \text{or} \quad l = 100/5 = 20 \text{ mm} \quad \text{Ans.}$$

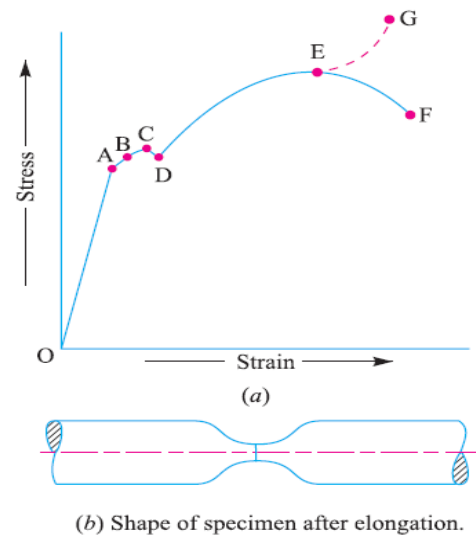
### 1.17 Stress-strain Diagram

In designing various parts of a machine, it is necessary to know how the material will function in service. For this, certain characteristics or properties of the material should be known. The mechanical properties mostly used in mechanical engineering practice are commonly determined from a standard tensile test. This test consists of gradually loading a standard specimen of a material and noting the corresponding values of load and elongation until the specimen fractures. The load is applied and measured by a testing machine. The stress is determined by dividing the load values by the original cross-sectional area of the specimen. The elongation is measured by determining the amounts that two reference points on the specimen are moved apart by the action of the machine. The original distance between the two reference points is known as gauge length. The strain is determined by dividing the elongation values by the gauge length. The values of the stress and corresponding strain are used to draw the stress-strain diagram of the material tested. A stress-strain diagram for a mild steel under tensile test is shown in Fig. (a). The various properties of the material are discussed below :

**1. Proportional limit.** We see from the diagram that from point O to A is a straight line, which represents that the stress is proportional to strain. Beyond point A, the curve slightly deviates from the straight line. It's thus obvious, that Hooke's law holds good up to point A and it is known as **proportional limit**. It is defined as that stress at which the stress-strain curve begins to deviate from the straight line.

**2. Elastic limit.** It may be noted that even if the load is increased beyond point A up to the point B, the material will regain its shape and size when the load is removed. This means that the material has elastic properties up to the point B. This point is known as **elastic limit**. It is defined as the stress developed in the material without any permanent set.

**Note:** Since the above two limits are very close to each other, therefore, for all practical purposes these are taken to be equal.





**3. Yield point.** If the material is stressed beyond point B, the plastic stage will reach i.e. on the removal of the load, the material will not be able to recover its original size and shape. A little consideration will show that beyond point B, the strain increases at a faster rate with any increase in the stress until the point C is reached. At this point, the material yields before the load and there is an appreciable strain without any increase in stress. In case of mild steel, it will be seen that a small load drops to D, immediately after yielding commences. Hence there are two yield points C and D. The points C and D are called the **upper** and **lower yield points** respectively. The stress corresponding to yield point is known as **yield point stress**.

**4. Ultimate stress.** At D, the specimen regains some strength and higher values of stresses are required for higher strains, than those between A and D. The stress (or load) goes on increasing till the point E is reached. The gradual increase in the strain (or length) of the specimen is followed with the uniform reduction of its cross-sectional area. The work done, during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At E, the stress, which attains its maximum value is known as **ultimate stress**. It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

**5. Breaking stress.** After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross-sectional area of the specimen, as shown in Fig. (b). A little consideration will show that the stress (or load) necessary to break away the specimen, is less than the maximum stress. The stress is, therefore, reduced until the specimen breaks away at point F. The stress corresponding to point F is known as **breaking stress**.

**Note :** *The breaking stress (i.e. stress at F which is less than at E) appears to be somewhat misleading. As the formation of a neck takes place at E which reduces the cross-sectional area, it causes the specimen suddenly to fail at F. If for each value of the strain between E and F, the tensile load is divided by the reduced cross-sectional area at the narrowest part of the neck, then the true stress-strain curve will follow the dotted line EG. However, it is an established practice, to calculate strains on the basis of original cross-sectional area of the specimen.*

**6. Percentage reduction in area.** It is the difference between the original cross-sectional area and cross-sectional area at the neck (i.e. where the fracture takes place). This difference is expressed as percentage of the original cross-sectional area.

Let  $A$  = Original cross-sectional area, and  
 $a$  = Cross-sectional area at the neck.

Then reduction in area =  $A - a$

And percentage reduction in area =  $(A - a / A) \times 100$

**7. Percentage elongation.** It is the percentage increase in the standard gauge length (i.e. original length) obtained by measuring the fractured specimen after bringing the broken parts together.

Let  $l$  = Gauge length or original length, and  
 $L$  = Length of specimen after fracture or final length.

Elongation =  $L - l$

and percentage elongation =  $(L - l / l) \times 100$

**Note :** *The percentage elongation gives a measure of ductility of the metal under test. The amount of local extensions depends upon the material and also on the transverse dimensions of the test piece. Since the specimens are to be made from bars, strips, sheets, wires, forgings, castings, etc., therefore it is not possible to make all specimens of one standard size. Since the dimensions of the specimen influence the result, therefore some standard means of comparison of results are necessary.*

As a result of series of experiments, Barba established a law that in tension, similar test pieces deform similarly and two test pieces are said to be similar if they have the same value of  $l / \sqrt{A}$ , where  $l$  is the gauge length and  $A$  is the cross-sectional area. A little consideration will show that the same material will give the same percentage elongation and percentage reduction in area.

It has been found experimentally by Unwin that the general extension (up to the maximum load) is proportional to the gauge length of the test piece and that the local extension (from maximum load to the breaking load) is proportional to the square root of the cross-sectional area. According to Unwin's formula, the increase in length,

$$\delta l = b.l + C \sqrt{A}$$

and percentage elongation =  $(\delta l / l) \times 100$

where

$l$  = Gauge length,

$A$  = Cross-sectional area, and

$b$  and  $C$  = Constants depending upon the quality of the material.

The values of  $b$  and  $C$  are determined by finding the values of  $\delta l$  for two test pieces of known length ( $l$ ) and area ( $A$ ).

**Example:** A mild steel rod of 12 mm diameter was tested for tensile strength with the gauge length of 60 mm. Following observations were recorded :

Final length = 80 mm; Final diameter = 7 mm; Yield load = 3.4 kN and Ultimate load = 6.1 kN.

Calculate : 1. yield stress, 2. ultimate tensile stress, 3. percentage reduction in area, and 4. percentage elongation.

**Solution.** Given :  $D = 12$  mm ;  $l = 60$  mm ;  $L = 80$  mm ;  $d = 7$  mm ;  $W_y = 3.4$  kN = 3400 N;

$W_u = 6.1$  kN = 6100 N

We know that original area of the rod,  $A = (\pi / 4) \times D^2 = (\pi / 4) \times (12)^2 = 113$  mm<sup>2</sup>  
and final area of the rod,  $a = (\pi / 4) \times d^2 = (\pi / 4) \times (7)^2 = 38.5$  mm<sup>2</sup>

### 1. Yield stress

We know that yield stress =  $W_y / A = 3400 / 113 = 30.1$  N/mm<sup>2</sup> = 30.1 MPa Ans.

### 2. Ultimate tensile stress

We know the ultimate tensile stress =  $W_u / A = 6100 / 113 = 54$  N/mm<sup>2</sup> = 54 MPa Ans.

### 3. Percentage reduction in area

We know that percentage reduction in area =  $(A - a) / A = (113 - 38.5) / 113 = 0.66$  or 66% Ans.

### 4. Percentage elongation

We know that percentage elongation =  $(L - l) / l = (80 - 60) / 60 = 0.33$  or 33% Ans.

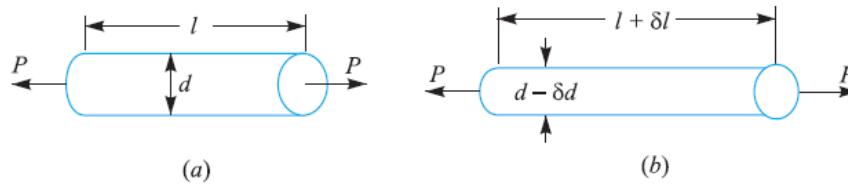
### 1.18 Working Stress:

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the **working stress** or **design stress**. It is also known as **safe** or **allowable stress**.

**Note :** By failure it is not meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactory.

### 1.19 Linear and Lateral Strain:

Consider a circular bar of diameter  $d$  and length  $l$ , subjected to a tensile force  $P$  as shown in Fig. (a).



A little consideration will show that due to tensile force, the length of the bar increases by an amount  $\delta l$  and the diameter decreases by an amount  $\delta d$ , as shown in Fig. (b). Similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as **linear strain** and an opposite kind of strain in every direction, at right angles to it, is known as **lateral strain**.

### 1.20 Poisson's Ratio:

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\text{Lateral strain} / \text{Linear strain} = \text{Constant}$$

This constant is known as Poisson's ratio and is denoted by  $1/m$  or  $\mu$ .

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

**Table 1.4. Values of Poisson's ratio for commonly used materials.**

<u>S.No.</u>	<u>Material</u>	<u>Poisson's ratio (1/m or <math>\mu</math>)</u>
1	Steel	0.25 to 0.33
2	Cast iron	0.23 to 0.273
3	Copper	0.31 to 0.34
4	Brass	0.32 to 0.42
5	Aluminium	0.32 to 0.36
6	Concrete	0.08 to 0.18
7	Rubber	0.45 to 0.50

### 1.21 Volumetric Strain:

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as volumetric strain. Mathematically, volumetric strain,

$$\epsilon_v = \delta V / V$$

where

$\delta V$  = Change in volume, and

$V$  = Original volume.

**Notes :** **1.** Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \delta V / V = \varepsilon (1 - (2/m)) ; \text{ where } \varepsilon = \text{Linear strain.}$$

**2.** Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where  $\varepsilon_x$ ,  $\varepsilon_y$  and  $\varepsilon_z$  are the strains in the directions x-axis, y-axis and z-axis respectively.

### 1.22 Bulk Modulus:

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as bulk modulus. It is usually denoted by K. Mathematically, bulk modulus,

$$K = \text{Direct stress} / \text{Volumetric strain} = \sigma / (\delta V / V)$$

### 1.23 Relation Between Bulk Modulus and Young's Modulus:

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = m.E / (3(m-2)) = E / (3(1-2\mu))$$

### 1.24 Relation Between Young's Modulus and Modulus of Rigidity:

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = m.E / (2(m+1)) = E / (2(1+\mu))$$

**Example:** A mild steel rod supports a tensile load of 50 kN. If the stress in the rod is limited to 100 MPa, find the size of the rod when the cross-section is 1. circular, 2. square, and 3. rectangular with width = 3 × thickness.

**Solution.** Given :  $P = 50 \text{ kN} = 50 \times 10^3 \text{ N}$  ;  $\sigma_t = 100 \text{ MPa} = 100 \text{ N/mm}^2$

#### 1. Size of the rod when it is circular

Let  $d = \text{Diameter of the rod in mm.}$

$$\text{Area, } A = (\pi/4) \times d^2 = 0.7854d^2$$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times 0.7854 d^2 = 78.54 d^2$$

$$d^2 = 50 \times 10^3 / 78.54 = 636.6 \text{ or } d = 25.23 \text{ mm}$$

Ans.

#### 2. Size of the rod when it is square

Let  $x = \text{Each side of the square rod in mm.}$

$$\text{Area, } A = x \times x = x^2$$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times x^2$$

$$x^2 = 50 \times 10^3 / 100 = 500 \text{ or } x = 22.4 \text{ mm}$$

Ans.

**3. Size of the rod when it is rectangular**

Let  $t =$  Thickness of the rod in mm, and  
 $b =$  Width of the rod in mm  $= 3t \dots$ (Given)

Area,  $A = b \times t = 3t \times t = 3t^2$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times 3t^2 = 300t^2$$

$$t^2 = 50 \times 10^3 / 300 = 166.7 \text{ or } t = 12.9 \text{ mm}$$

Ans.

And  $b = 3t = 3 \times 12.9 = 38.7 \text{ mm}$

Ans.

**1.25 Resilience:**

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as **strain energy**. The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as **resilience** and the maximum energy which can be stored in a body up to the elastic limit is called **proof resilience**. The proof resilience per unit volume of a material is known as **modulus of resilience**. It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy stored in a body due to tensile or compressive load or resilience,

$$U = \sigma^2 \times V / 2E$$

and Modulus of resilience  $= \sigma^2 / 2E$

where  $\sigma =$  Tensile or compressive stress,

$V =$  Volume of the body, and

$E =$  Young's modulus of the material of the body.

**Notes : 1.** When a body is subjected to a shear load, then modulus of resilience (shear)

$$= \tau^2 / 2G$$

Where  $\tau =$  Shear stress, and

$G =$  Modulus of rigidity.

**2.** When the body is subjected to torsion, then modulus of resilience

$$= \tau^2 / 4G$$

**Example:** A wrought iron bar 50 mm in diameter and 2.5 m long transmits a shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take  $E = 200 \text{ GN/m}^2$ .

**Solution.** Given :  $d = 50 \text{ mm}$  ;  $l = 2.5 \text{ m} = 2500 \text{ mm}$  ;  $U = 100 \text{ N-m} = 100 \times 10^3 \text{ N-mm}$  ;  $E = 200 \text{ GN/m}^2 = 200 \times 10^3 \text{ N/mm}^2$

**Maximum instantaneous stress**

Let  $\sigma =$  Maximum instantaneous stress.

We know that volume of the bar,

$$V = (\pi/4) \times d^2 \times l = (\pi/4) \times (50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \sigma^2 \times V / 2E = (\sigma^2 \times 4.9 \times 10^6) / (2 \times 200 \times 10^3) = 12.25 \sigma^2$$

$$\sigma^2 = 100 \times 10^3 / 12.25 = 8163 \text{ or } \sigma = 90.3 \text{ N/mm}^2$$

Ans.

**Elongation produced**

Let  $\delta l =$  Elongation produced.

We know that Young's modulus,

$$E = \text{Stress} / \text{Strain} = \sigma / \epsilon = \sigma l / \delta l$$

$$\delta l = \sigma l / E = (90.3 \times 2500) / (200 \times 10^3) = 1.13 \text{ mm}$$

Ans.

**1.26 Cartesian Stress Components:**

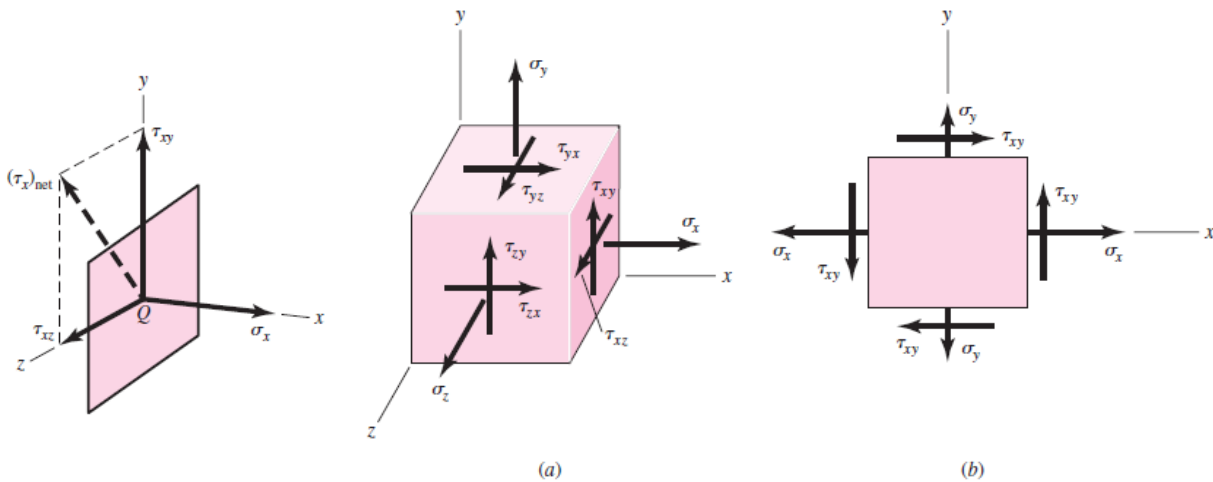


Fig.1. Stress components on surface normal to x direction.

Fig.2 (a) General three-dimensional stress.  
(b) Plane stress with "cross-shears" equal.

The Cartesian stress components are established by defining three mutually orthogonal surfaces at a point within the body. The normals to each surface will establish the x, y, z Cartesian axes. In general, each surface will have a normal and shear stress. The shear stress may have components along two Cartesian axes. For example, Fig. 1 shows an infinitesimal surface area isolation at a point Q within a body where the surface normal is the x direction. The normal stress is labeled  $\sigma_x$ . The symbol  $\sigma$  indicates a normal stress and the subscript x indicates the direction of the surface normal. The net shear stress acting on the surface is  $(\tau_x)_{\text{net}}$  which can be resolved into components in the y and z directions, labeled as  $\tau_{xy}$  and  $\tau_{xz}$ , respectively (see Fig. 1). Note that double subscripts are necessary for the shear. The first subscript indicates the direction of the surface normal whereas the second subscript is the direction of the shear stress. The state of stress at a point described by three mutually perpendicular surfaces is shown in Fig. 2a. It can be shown through coordinate transformation that this is sufficient to determine the state of stress on any surface intersecting the point. As the dimensions of the cube in Fig. 2a approach zero, the stresses on the hidden faces become equal and opposite to those on the opposing visible faces. Thus, in general, a complete state of stress is defined by nine stress components,  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$ ,  $\tau_{xy}$ ,  $\tau_{xz}$ ,  $\tau_{yx}$ ,  $\tau_{yz}$ ,  $\tau_{zx}$ , and  $\tau_{zy}$ .

For equilibrium, in most cases, “cross-shears” are equal, hence

$$\tau_{yx} = \tau_{xy}; \quad \tau_{zy} = \tau_{yz}; \quad \tau_{xz} = \tau_{zx} \quad (1-1)$$

This reduces the number of stress components for most three-dimensional states of stress from nine to six quantities,  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$ ,  $\tau_{xy}$ ,  $\tau_{yz}$ , and  $\tau_{zx}$ .

A very common state of stress occurs when the stresses on one surface are zero. When this occurs the state of stress is called plane stress. Figure 2b shows a state of plane stress, arbitrarily assuming that the normal for the stress-free surface is the z direction such that  $\sigma_z = \tau_{zx} = \tau_{zy} = 0$ . It is important to note that the element in Fig. 2b is still a three-dimensional cube. Also, here it is assumed that the cross-shears are equal such that  $\tau_{yx} = \tau_{xy}$ , and  $\tau_{yz} = \tau_{zy} = \tau_{xz} = \tau_{zx} = 0$ .

### 1-27 Mohr's Circle for Plane Stress:

Suppose the  $d_x d_y d_z$  element of Fig. 2b above is cut by an oblique plane with a normal n at an arbitrary angle  $\phi$  counterclockwise from the x axis as shown in Fig. This section is concerned with the stresses  $\sigma$  and  $\tau$  that act upon this oblique plane. By summing the forces caused by all the stress components

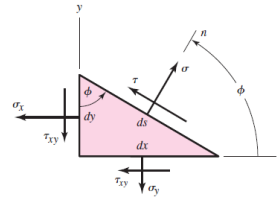
$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi \quad (1-2)$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi \quad (1-3)$$



Equations (1-2) and (1-3) are called the plane-stress transformation equations. Differentiating Eq. (1-2) with respect to  $\phi$  and setting the result equal to zero gives

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad (1-4)$$



Equation (1-4) defines two particular values for the angle  $2\phi_p$ , one of which defines the maximum normal stress  $\sigma_1$  and the other, the minimum normal stress  $\sigma_2$ . These two stresses are called the principal stresses, and their corresponding directions, the principal directions. The angle between the principal directions is  $90^\circ$ . It is important to note that Eq. (1-4) can be written in the form

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\phi_p - \tau_{xy} \cos 2\phi_p = 0 \quad (a)$$

Comparing this with Eq. (1-3), we see that  $\tau=0$ , meaning that the surfaces containing principal stresses have zero shear stresses. In a similar manner, we differentiate Eq. (1-3), set the result equal to zero, and obtain:

$$\tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \quad (1-5)$$

Equation (1-5) defines the two values of  $2\phi_s$  at which the shear stress  $\tau$  reaches an extreme value. The angle between the surfaces containing the maximum shear stresses is  $90^\circ$ . Equation (1-5) can also be written as:

$$\frac{\sigma_x - \sigma_y}{2} \cos 2\phi_p + \tau_{xy} \sin 2\phi_p = 0 \quad (b)$$

Substituting this into Eq. (1-2) yields:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \quad (1-6)$$

Equation (1-6) tells us that the two surfaces containing the maximum shear stresses also contain equal normal stresses of  $(\sigma_x + \sigma_y)/2$ .

Comparing Eqs. (1-4) and (1-5), we see that  $\tan 2\phi_s$  is the negative reciprocal of  $\tan 2\phi_p$ . This means that  $2\phi_s$  and  $2\phi_p$  are angles  $90^\circ$  apart, and thus the angles between the surfaces containing the maximum shear stresses and the surfaces containing the principal stresses are  $\pm 45^\circ$ .

Formulas for the two principal stresses can be obtained by substituting the angle  $2\phi_p$  from Eq. (1-4) in Eq. (1-2). The result is :

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (1-7)$$

In a similar manner the two extreme-value shear stresses are found to be:

$$\tau_1, \tau_2 = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (1-8)$$

***This convention is followed in drawing Mohr's circle:***

- Shear stresses tending to rotate the element clockwise (cw) are plotted above the  $\sigma$  axis.
- Shear stresses tending to rotate the element counterclockwise (ccw) are plotted below the  $\sigma$  axis.

For example, consider the right face of the element in Fig. 2b. By Mohr's circle convention the shear stress shown is plotted below the  $\sigma$  axis because it tends to rotate the element counterclockwise. The shear stress on the top face of the element is plotted above the  $\sigma$  axis because it tends to rotate the element clockwise.

In Figure 3-10 pg 79 we create a coordinate system with normal stresses plotted along the abscissa and shear stresses plotted as the ordinates. On the abscissa, tensile (positive) normal stresses are plotted to the right of the origin O and compressive (negative) normal stresses to the left. On the ordinate, clockwise (cw) shear stresses are plotted up; counterclockwise (ccw) shear stresses are plotted down.

***1.28 Normal Stresses for Beams in Bending:***

The equations for the normal bending stresses in straight beams are based on the following assumptions:

1. The beam is subjected to pure bending. This means that the shear force is zero, and that no torsion or axial loads are present.
2. The material is isotropic and homogeneous.
3. The material obeys Hooke's law.
4. The beam is initially straight with a cross section that is constant throughout the beam length.
5. The beam has an axis of symmetry in the plane of bending.
6. The proportions of the beam are such that it would fail by bending rather than by crushing, wrinkling, or sidewise buckling.
7. Plane cross sections of the beam remain plane during bending.

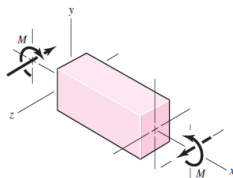


Fig. Straight beam in positive bending.

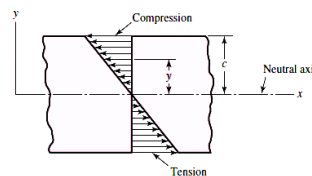


Fig. Bending stresses according to Eq. (1-9).

The bending stress varies linearly with the distance from the neutral axis,  $y$ , and is given by:

$$\sigma_x = -\frac{My}{I} \quad (1-9)$$

where  $I$  is :

$$I = \int y^2 dA \quad (1-10)$$

The maximum magnitude of the bending stress will occur where  $y$  has the greatest magnitude. Designating  $\sigma_{\max}$  as the maximum magnitude of the bending stress, and  $c$  as the maximum magnitude of  $y$ :

$$\sigma_{\max} = \frac{Mc}{I} \quad (1-11)$$

### 1.29 Two-Plane Bending:

Quite often, in mechanical design, bending occurs in both  $xy$  and  $xz$  planes. Considering cross sections with one or two planes of symmetry only, the bending stresses are given by:

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \quad (1-12)$$

where the first term on the right side of the equation is identical to  $\sigma = M.Y/I$ ,  $M_y$  is the bending moment in the  $xz$  plane (moment vector in  $y$  direction),  $z$  is the distance from the neutral  $y$  axis, and  $I_y$  is the second area moment about the  $y$  axis.

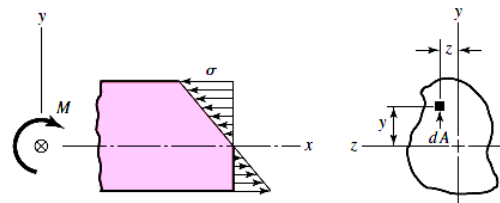
For noncircular cross sections, Eq. (1-12) is the superposition of stresses caused by the two bending moment components. The maximum tensile and compressive bending stresses occur where the summation gives the greatest positive and negative stresses, respectively. For solid circular cross sections, all lateral axes are the same and the plane containing the moment corresponding to the vector sum of  $M_z$  and  $M_y$  contains the maximum bending stresses. For a beam of diameter  $d$  the maximum distance from the neutral axis is  $d/2$ . The maximum bending stress for a solid circular cross section is then:

$$\sigma_m = \frac{Mc}{I} = \frac{(M_y^2 + M_z^2)^{1/2}(d/2)}{\pi d^4/64} = \frac{32}{\pi d^3}(M_y^2 + M_z^2)^{1/2} \quad (1-13)$$

### 1.30 Shear Stresses for Beams in Bending:

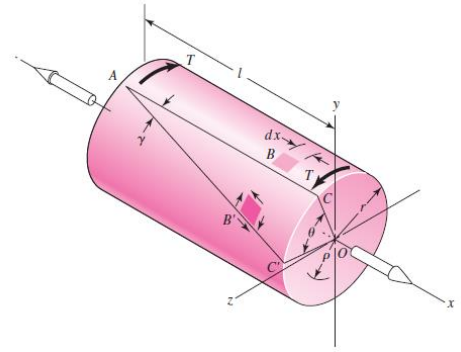
$$\tau = \frac{V}{Ib} \int_{y_1}^c y dA$$

$$\tau = \frac{VQ}{Ib}$$



**1.31 Torsion:**

Any moment vector that is collinear with an axis of a mechanical element is called a torque vector, because the moment causes the element to be twisted about that axis. A bar subjected to such a moment is also said to be in torsion. As shown in Figure, the torque  $T$  applied to a bar can be designated by drawing arrows on the surface of the bar to indicate direction or by drawing torque-vector arrows along the axes of twist of the bar.



Torque vectors are the hollow arrows shown on the  $x$  axis in figure above. Note that they conform to the right-hand rule for vectors. The angle of twist, in radians, for a solid round bar is:

$$\theta = \frac{Tl}{GJ} \quad (1-14)$$

where  $T$  = torque.

$l$  = length.

$G$  = modulus of rigidity.

$J$  = polar second moment of area

Shear stresses develop throughout the cross section. For a round bar in torsion, these stresses are proportional to the radius  $\rho$  and are given by:

$$\tau = \frac{T\rho}{J} \quad (1-15)$$

Designating  $r$  as the radius to the outer surface, we have:

$$\tau_{\max} = \frac{Tr}{J} \quad (1-16)$$

The assumptions used in the analysis are:

- The bar is acted upon by a pure torque, and the sections under consideration are remote from the point of application of the load and from a change in diameter.
- Adjacent cross sections originally plane and parallel remain plane and parallel after twisting, and any radial line remains straight.
- The material obeys Hooke's law.

Equation (1-16) applies only to circular sections. For a solid round section,  $J = \frac{\pi d^4}{32}$  (1-17)

where  $d$  is the diameter of the bar. For a hollow round section,  $J = \frac{\pi}{32}(d_o^4 - d_i^4)$  (1-18)

where the subscripts  $o$  and  $i$  refer to the outside and inside diameters, respectively.

In using Eq. ( 1-16 ) it is often necessary to obtain the torque T from a consideration of the power and speed of a rotating shaft. For convenience when U. S. Customary units are used, three forms of this relation are:

$$H = \frac{FV}{33\,000} = \frac{2\pi Tn}{33\,000(12)} = \frac{Tn}{63\,025} \quad (1-19)$$

where H = power, hp. T = torque, lbf · in.  
 n = shaft speed, rev/min. F = force, lbf.  
 V = velocity, ft/min

When SI units are used, the equation is:  $H = T\omega$  (1-20)

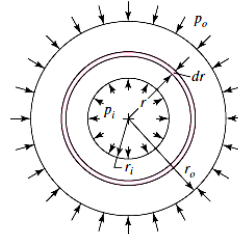
Where H = power in Watt.  
 T = torque in N · m.  
 $\omega$  = angular velocity in rad/s

### 1.32 Stresses in thick Cylinders:

Cylindrical pressure vessels, hydraulic cylinders, gun barrels, and pipes carrying fluids at high pressures develop both radial and tangential stresses with values that depend upon the radius of the element under consideration. In determining the radial stress  $\sigma_r$  and the tangential stress  $\sigma_t$ , we make use of the assumption that the longitudinal elongation is constant around the circumference of the cylinder. In other words, a right section of the cylinder remains plane after stressing.

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i) / r^2}{r_o^2 - r_i^2} \quad (1-21)$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i) / r^2}{r_o^2 - r_i^2} \quad (1-22)$$



As usual, positive values indicate tension and negative values, compression. The special case of  $p_o = 0$  gives :

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2} \left( 1 - \frac{r_o^2}{r^2} \right) \quad (1-23)$$

It should be realized that longitudinal stresses exist when the end reactions to the internal pressure are taken by the pressure vessel itself. This stress is found to be :

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} \quad (1-24)$$

### 1.33 Thin-Walled Vessels.

When the wall thickness of a cylindrical pressure vessel is about one-twentieth, or less, of its radius, the radial stress that results from pressurizing the vessel is quite small compared with the tangential stress. Under these conditions the tangential stress can be obtained as follows: Let an internal pressure  $p$  be exerted on the wall of a cylinder of thickness  $t$  and inside diameter  $d_i$ .

The force tending to separate two halves of a unit length of the cylinder is  $p d_i$ . This force is resisted by the tangential stress, also called the hoop stress, acting uniformly over the stressed area. We then have  $p d_i = 2t\sigma_t$ , or:

$$(\sigma_t)_{av} = \frac{p d_i}{2t} \quad (1-25)$$

This equation gives the average tangential stress and is valid regardless of the wall thickness.

For a thin-walled vessel an approximation to the maximum tangential stress is:

$$(\sigma_t)_{max} = \frac{p(d_i + t)}{2t} \quad (1-26)$$

where  $d_i + t$  is the average diameter.

In a closed cylinder, the longitudinal stress  $\sigma_l$  exists because of the pressure upon the ends of the vessel. If we assume this stress is also distributed uniformly over the wall thickness, we can easily find it to be:

$$\sigma_l = \frac{p d_i}{4t} \quad (1-27)$$

### 1.34 Temperature Effects:

When the temperature of an unrestrained body is uniformly increased, the body expands, and the normal strain is :

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha(\Delta T) \quad (1-28)$$

where  $\alpha$  is the coefficient of thermal expansion and  $\Delta T$  is the temperature change, in degrees. In this action the body experiences a simple volume increase with the components of shear strain all zero. If a straight bar is restrained at the ends so as to prevent lengthwise expansion and then is subjected to a uniform increase in temperature, a compressive stress will develop because of the axial constraint. The stress is

$$\sigma = -\epsilon E = -\alpha(\Delta T)E \quad (1-29)$$

In a similar manner, if a uniform flat plate is restrained at the edges and also subjected to a uniform temperature rise, the compressive stress developed is given by the equation:

$$\sigma = -\frac{\alpha(\Delta T)E}{1 - \nu} \quad (1-30)$$

### 1.35 Curved Beams in Bending:

The distribution of stress in a curved flexural member is determined by using the following assumptions:

- The cross section has an axis of symmetry in a plane along the length of the beam.
- Plane cross sections remain plane after bending.
- The modulus of elasticity is the same in tension as in compression.

We shall find that the neutral axis and the centroidal axis of a curved beam, unlike the axes of a straight beam, are not coincident and also that the stress does not vary linearly from the neutral axis. The notation shown in Figure below is defined as follows:

$r_o$  = radius of outer fiber

$r_i$  = radius of inner fiber

$h$  = depth of section

$c_o$  = distance from neutral axis to outer fiber

$c_i$  = distance from neutral axis to inner fiber

$r_n$  = radius of neutral axis

$r_c$  = radius of centroidal axis

$e$  = distance from centroidal axis to neutral axis

$M$  = bending moment; positive  $M$  decreases curvature.

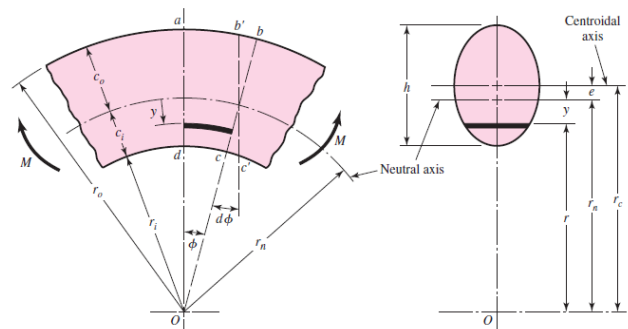


Figure above shows that the neutral and centroidal axes are not coincident. It turns out that the location of the neutral axis with respect to the center of curvature  $O$  is given by the equation:

$$r_n = \frac{A}{\int \frac{dA}{r}} \quad (1-31)$$

The stress distribution can be found by balancing the external applied moment against the internal resisting moment. The result is found to be:

$$\sigma = \frac{My}{Ae(r_n - y)} \quad (1-32)$$

where  $M$  is positive in the direction shown in Figure above. Equation (1-33) shows that the stress distribution is hyperbolic. The critical stresses occur at the inner and outer surfaces where  $y = c_i$  and  $y = -c_o$ , respectively, and are:

$$\sigma_i = \frac{Mc_i}{Aer_i} \quad \sigma_o = -\frac{Mc_o}{Aer_o} \quad (1-33)$$

**1.36 Deflection Due to Bending:**

The problem of bending of beams probably occurs more often than any other loading problem in mechanical design. Shafts, axles, cranks, levers, springs, brackets, and wheels, as well as many other elements, must often be treated as beams in the design and analysis of mechanical structures and systems. The subject of bending, however, is one that you should have studied as preparation for reading this book. It is for this reason that we include here only a brief review to establish the nomenclature and conventions to be used throughout this book. The curvature of a beam subjected to a bending moment  $M$  is given by:

$$\frac{1}{\rho} = \frac{M}{EI} \quad (1-34)$$

where  $\rho$  is the radius of curvature. From studies in mathematics we also learn that the curvature of a plane curve is given by the equation:

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{[1 + (dy/dx)^2]^{3/2}} \quad (1-35)$$

where the interpretation here is that  $y$  is the lateral deflection of the beam at any point  $x$  along its length.

The slope of the beam at any point  $x$  is:

$$\theta = \frac{dy}{dx} \quad (a)$$

For many problems in bending, the slope is very small, and for these the denominator of Eq. (1-35) can be taken as unity. Equation (1-34) can then be written:

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \quad (b)$$

Noting Eqs.  $V = \frac{dM}{dx}$  and  $\frac{dV}{dx} = \frac{d^2M}{dx^2} = q$  and successively differentiating Eq. (b) yields:

$$\frac{V}{EI} = \frac{d^3y}{dx^3} \quad (c) \quad \text{and} \quad \frac{q}{EI} = \frac{d^4y}{dx^4} \quad (d)$$

It is convenient to display these relations in a group as follows:

$$\frac{q}{EI} = \frac{d^4y}{dx^4} \quad (1-36)$$

$$\frac{V}{EI} = \frac{d^3y}{dx^3} \quad (1-37)$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \quad (1-38)$$

$$\theta = \frac{dy}{dx} \quad (1-39)$$

$$y = f(x) \quad (1-40)$$



## CHAPTER TWO

### Failures Resulting from Static Loading

A *static load* is a stationary force or couple applied to a member. To be stationary, the force or couple must be unchanging in magnitude, point or points of application, and direction. A static load can produce axial tension or compression, a shear load, a bending load, a torsional load, or any combination of these. To be considered static, the load cannot change in any manner.

The relations between strength and static loading will consider in order to make the decisions concerning material and its treatment, fabrication, and geometry for satisfying the requirements of functionality, safety, reliability, competitiveness, usability, manufacturability, and marketability.

“*Failure*” is the first word in the chapter title. Failure can mean a part has separated into two or more pieces; has become permanently distorted, thus ruining its geometry. A designer speaking of failure can mean any or all of these possibilities. In this chapter our attention is focused on the predictability of permanent distortion or separation. In strength-sensitive situations the designer must separate mean stress and mean strength at the critical location sufficiently to accomplish his or her purposes.

In designing part to resist failure we usually assume our solves that the internal stresses do not exceed the strength of the material. *For the ductile material*, the yield strength are usually interested in due to a permanent deformation would constitute failure. While *the brittle materials* do not have a yield point, thus the ultimate strength as the criterion of failure must applied in design. In designing parts of brittle materials it is also necessary to remember that the ultimate compressive strength is much greater than the ultimate tensile strength. But the strength of ductile materials are the same in both tension and compression.

You can now appreciate the following four design categories:

1. Failure of the part would endanger human life, or the part is made in extremely large quantities; consequently, an elaborate testing program is justified during design.
2. The part is made in large enough quantities that a moderate series of tests is feasible.
3. The part is made in such small quantities that testing is not justified at all; or the design must be completed so rapidly that there is not enough time for testing.
4. The part has already been designed, manufactured, and tested and found to be unsatisfactory. Analysis is required to understand why the part is unsatisfactory and what to do to improve it.

## 2.1 Stress Concentration

Stress concentration is a highly localized effect. In some instances it may be due to a surface scratch. If the material is ductile and the load static, the design load may cause yielding in the critical location in the notch. This yielding can involve strain strengthening of the material and an increase in yield strength at the small critical notch location. Since the loads are static and the material is ductile, that part can carry the loads satisfactorily with no general yielding. In these cases the designer sets the geometric (theoretical) stress-concentration factor  $K_t$  to unity. The usual definition of geometric (theoretical) stress concentration factor for normal stress  $K_t$  and shear stress  $K_{ts}$  is given by Eq. as

$$\sigma_{\max} = K_t \sigma_{\text{nom}} \quad (\text{a})$$

$$\tau_{\max} = K_{ts} \tau_{\text{nom}} \quad (\text{b})$$

Since your attention is on the stress-concentration factor, and the definition of  $\sigma_{\text{nom}}$  or  $\tau_{\text{nom}}$  is given in the graph caption or from a computer program, be sure the value of nominal stress is appropriate for the section carrying the load.

**Note:** When using this rule for ductile materials with static loads, be careful to assure yourself that the material is not susceptible to brittle fracture in the environment of use.

## 2.2 Failure Theories:

Structural metal behavior is typically classified as being ductile or brittle, although under special situations, a material normally considered ductile can fail in a brittle manner. Ductile materials are normally classified such that  $\epsilon_f \geq 0.05$  and have an identifiable yield strength that is often the same in compression as in tension ( $S_{yt} = S_{yc} = S_y$ ). Brittle materials,  $\epsilon_f < 0.05$ , do not exhibit an identifiable yield strength, and are typically classified by ultimate tensile and compressive strengths,  $S_{ut}$  and  $S_{uc}$ , respectively (where  $S_{uc}$  is given as a positive quantity). The generally accepted theories are:

### **Ductile materials (yield criteria)**

- Maximum shear stress (MSS)
- Distortion energy (DE)
- Ductile Coulomb-Mohr (DCM)

### **Brittle materials (fracture criteria)**

- Maximum normal stress (MNS)
- Brittle Coulomb-Mohr (BCM)
- Modified Mohr (MM),

### 2.3 Maximum-Shear-Stress Theory for Ductile Materials:

The maximum-shear-stress theory predicts that yielding begins whenever the maximum shear stress in any element equals or exceeds the maximum shear stress in a tension test specimen of the same material when that specimen begins to yield. The MSS theory is also referred to as the *Tresca* or Guest theory.

Recall that for simple tensile stress,  $\sigma = P/A$ , and the maximum shear stress occurs on a surface ( $45^\circ$ ) from the tensile surface with a magnitude of  $\tau_{\max} = \sigma/2$ . So the maximum shear stress at yield is  $\tau_{\max} = S_y/2$ . For a general state of stress, three principal stresses can be determined and ordered such that  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ . The maximum shear stress is then  $\tau_{\max} = (\sigma_1 - \sigma_3)/2$ . Thus, for a general state of stress, the maximum-shear-stress theory predicts yielding when

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \quad \text{or} \quad \sigma_1 - \sigma_3 \geq S_y \quad (2-1)$$

Note that this implies that the yield strength in shear is given by:

$$S_{sy} = 0.5S_y \quad (2-2)$$

For design purposes, Eq. (2-1) can be modified to incorporate a factor of safety,  $n$ . Thus:

$$\tau_{\max} = \frac{S_y}{2n} \quad \text{or} \quad \sigma_1 - \sigma_3 = \frac{S_y}{n} \quad (2-3)$$

Plane stress problems are very common where one of the principal stresses is zero, and the other two,  $\sigma_A$  and  $\sigma_B$ , are determined from principle Equation. Assuming that  $\sigma_A \geq \sigma_B$ , there are three cases to consider in using Eq. (2-1) for plane stress:

**Case 1:**  $\sigma_A \geq \sigma_B \geq 0$ . For this case,  $\sigma_1 = \sigma_A$  and  $\sigma_3 = 0$ . Equation (2-1) reduces to a yield condition of

$$\sigma_A \geq S_y \quad (2-4)$$

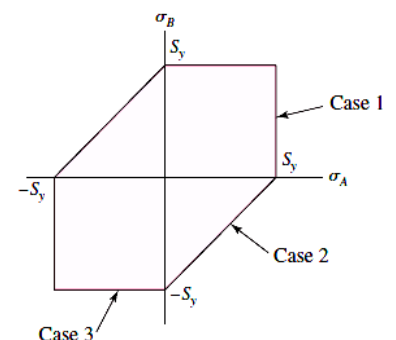
**Case 2:**  $\sigma_A \geq 0 \geq \sigma_B$ . Here,  $\sigma_1 = \sigma_A$  and  $\sigma_3 = \sigma_B$ , and Equation (2-1) becomes

$$\sigma_A - \sigma_B \geq S_y \quad (2-5)$$

**Case 3:**  $0 \geq \sigma_A \geq \sigma_B$ . For this case,  $\sigma_1 = 0$  and  $\sigma_3 = \sigma_B$ , and Equation (2-1) gives

$$\sigma_B \leq -S_y \quad (2-6)$$

Equations (2-4) to (2-6) are represented in Figure below by the three lines indicated in the  $\sigma_A, \sigma_B$  plane. The remaining unmarked lines are cases for  $\sigma_B \geq \sigma_A$ , which completes the stress yield envelope but are not normally used. Equations (2-4) to (2-6) can also be converted to design equations by substituting equality for the equal to or greater sign and dividing  $S_y$  by  $n$ .



Note that the first part of Eq. (2-3),  $\tau_{\max} = S_y/2n$ , is sufficient for design purposes provided the designer is careful in determining  $\tau_{\max}$ . For plane stress, Equation below

$$\tau_1, \tau_2 = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

does not always predict  $\tau_{\max}$ . However, consider the special case when one normal stress is zero in the plane, say  $\sigma_x$  and  $\tau_{xy}$  have values and  $\sigma_y = 0$ . It can be easily shown that this is a Case 2 problem, and the shear stress determined by Equation above is  $\tau_{\max}$ . Shaft design problems typically fall into this category where a normal stress exists from bending and/or axial loading, and a shear stress arises from torsion

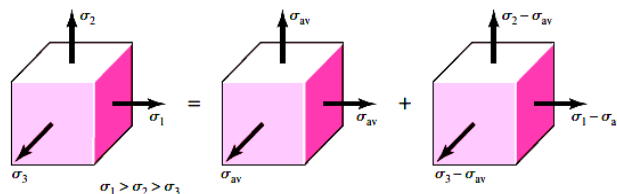
### 2.3 Distortion-Energy Theory for Ductile Materials:

The distortion-energy theory predicts that yielding occurs when the distortion strain energy per unit volume reaches or exceeds the distortion strain energy per unit volume for yield in simple tension or compression of the same material.

The distortion-energy (DE) theory originated from the observation that ductile materials stressed hydrostatically exhibited yield strengths greatly in excess of the values given by the simple tension test. Therefore it was postulated that yielding was not a simple tensile or compressive phenomenon at all, but, rather, that it was related somehow to the angular distortion of the stressed element. To develop the theory, note, in Fig. a, the unit volume subjected to any three-dimensional stress state designated by the stresses  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$ . The stress state shown in Fig. b is one of hydrostatic tension due to the stresses  $\sigma_{av}$  acting in each of the same principal directions as in Fig. a. The formula for  $\sigma_{av}$  is simply

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad (a)$$

Thus the element in Fig. b undergoes pure volume change, that is, no angular distortion. If we regard  $\sigma_{av}$  as a component of  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$ , then this component can be



(a) Triaxial stresses (b) Hydrostatic component (c) Distortional component

Fig. (a) Element with triaxial stresses; this element undergoes both volume change and angular distortion.

(b) Element under hydrostatic tension undergoes only volume change.

(c) Element has angular distortion without volume change.

subtracted from them, resulting in the stress state shown in Fig. c. This element is subjected to pure angular distortion, that is, no volume change. The strain energy per unit volume for simple tension is

$$u = (1/2) \epsilon \sigma$$

For the element of Fig. a the strain energy per unit volume is  $u = (1/2)[\epsilon_1 \sigma_1 + \epsilon_2 \sigma_2 + \epsilon_3 \sigma_3]$ . Substituting for the principal strains gives:

$$\begin{aligned} \epsilon_x &= \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] & \epsilon_1 &= \frac{1}{E} (\sigma_1 - \nu\sigma_2 - \nu\sigma_3) \\ \epsilon_y &= \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] & \text{or} & & \epsilon_2 &= \frac{1}{E} (\sigma_2 - \nu\sigma_1 - \nu\sigma_3) \\ \epsilon_z &= \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] & & & \epsilon_3 &= \frac{1}{E} (\sigma_3 - \nu\sigma_1 - \nu\sigma_2) \end{aligned}$$

$$u = \frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \quad (b)$$

The strain energy for producing only volume change  $u_v$  can be obtained by substituting  $\sigma_{av}$  for  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  in Eq. (b). The result is

$$u_v = \frac{3\sigma_{av}^2}{2E} (1 - 2\nu) \quad (c)$$

If we now substitute the square of Eq. (a) in Eq. (c) and simplify the expression, we get:

$$u_v = \frac{1 - 2\nu}{6E} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1) \quad (2-7)$$

Then the distortion energy is obtained by subtracting Eq. (2-7) from Eq. (b). This gives:

$$u_d = u - u_v = \frac{1 + \nu}{3E} \left[ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right] \quad (2-8)$$

Note that the distortion energy is zero if  $\sigma_1 = \sigma_2 = \sigma_3$ .

For the simple tensile test, at yield,  $\sigma_1 = S_y$  and  $\sigma_2 = \sigma_3 = 0$ , and from Eq. (2-8) the distortion energy is:

$$u_d = \frac{1 + \nu}{3E} S_y^2 \quad (2-9)$$

So for the general state of stress given by Eq. (2-8), yield is predicted if Eq. (2-8) equals or exceeds Eq.(2-9). This gives

$$\left[ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]^{1/2} \geq S_y \quad (2-10)$$

If we had a simple case of tension  $\sigma$ , then yield would occur when  $\sigma \geq S_y$ . Thus, the left of Eq. (2-10) can be thought of as a single, equivalent, or effective stress for the entire general state of stress given by  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$ . This effective stress is usually called the **von Mises stress**,  $\sigma'$ , named after Dr. R. von Mises, who contributed to the theory. Thus Eq. (2-10), for yield, can be written as:

$$\sigma' \geq S_y \quad (2-11)$$

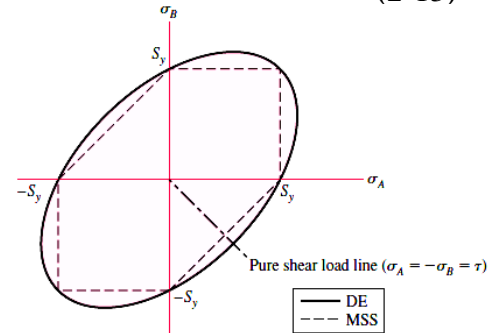
where the von Mises stress is:

$$\sigma' = \left[ \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]^{1/2} \quad (2-12)$$

For plane stress, let  $\sigma_A$  and  $\sigma_B$  be the two nonzero principal stresses. Then from Eq. (2-12), we get

$$\sigma' = (\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2)^{1/2} \quad (2-13)$$

Equation (2-13) is a rotated ellipse in the  $\sigma_A, \sigma_B$  plane, as shown in Fig. with  $\sigma' = S_y$ . The dotted lines in the figure represent the MSS theory, which can be seen to be more restrictive, hence, more conservative.



Using xyz components of three-dimensional stress, the von Mises stress can be written as:

$$\sigma' = \frac{1}{\sqrt{2}} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right]^{1/2} \quad (2-14)$$

and for plane stress:

$$\sigma' = (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2} \quad (2-15)$$

The mathematical manipulation involved in describing the DE theory might tend to obscure the real value and usefulness of the result. The equations given allow the most complicated stress situation to be represented by a single quantity, the von Mises stress, which then can be compared against the yield strength of the material through Eq. (2-11). This equation can be expressed as a design equation by

$$\sigma' = S_y / n \quad (2-16)$$

The distortion-energy theory predicts no failure under hydrostatic stress and agrees well with all data for ductile behavior. Hence, it is the most widely used theory for ductile materials and is recommended for design problems unless otherwise specified.

Thus, the shear yield strength predicted by the distortion-energy theory is

$$S_{sy} = 0.577S_y \quad (2-17)$$

which as stated earlier, is about 15 percent greater than the  $0.5 S_y$  predicted by the MSS theory.

### 2.4 Coulomb-Mohr Theory for Ductile Materials:

Not all materials have compressive strengths equal to their corresponding tensile values. For example, the yield strength of magnesium alloys in compression may be as little as 50 percent of their yield strength in tension. The ultimate strength of gray cast irons in compression varies from 3 to 4 times greater than the ultimate tensile strength. So, in this section, we are primarily interested in those theories that can be used to predict failure for materials whose strengths in tension and compression are not equal.

By simplifying, we can consider the following equation

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1 \quad (2-16)$$

where either *yield strength* **or** *ultimate strength* can be used.

For plane stress, when the two nonzero principal stresses are  $\sigma_A \geq \sigma_B$ , we have a situation similar to the three cases given for the MSS theory, Eqs. (2-4) to (2-6). That is, the failure conditions are:

**Case 1:**  $\sigma_A \geq \sigma_B \geq 0$ . For this case,  $\sigma_1 = \sigma_A$  and  $\sigma_3 = 0$ . Equation (2-16) reduces to a failure condition of

$$\sigma_A \geq S_t \quad (2-17)$$

**Case 2:**  $\sigma_A \geq 0 \geq \sigma_B$ . Here,  $\sigma_1 = \sigma_A$  and  $\sigma_3 = \sigma_B$ , and Eq. (2-16) becomes

$$\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1 \quad (2-18)$$

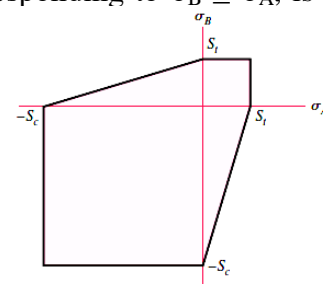
**Case 3:**  $0 \geq \sigma_A \geq \sigma_B$ . For this case,  $\sigma_1 = 0$  and  $\sigma_3 = \sigma_B$ , and Eq. (2-16) gives

$$\sigma_B \leq -S_c \quad (2-19)$$

A plot of these cases, together with the normally unused cases corresponding to  $\sigma_B \geq \sigma_A$ , is shown in Figure.

For design equations, incorporating the factor of safety  $n$ , divide all strengths by  $n$ . For example, Eq.(2-16) as a design equation can be written as:

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n} \quad (2-20)$$



Since for the Coulomb-Mohr theory we do not need the torsional shear strength circle we can deduce it from Eq.(2-16). For pure shear  $\tau$ ,  $\sigma_1 = -\sigma_3 = \tau$ . The torsional yield strength occurs when  $\tau_{\max} = S_{sy}$ . Substituting  $\sigma_1 = -\sigma_3 = S_{sy}$  into Eq. (2-16) and simplifying gives :

$$S_{sy} = \frac{S_{yt} S_{yc}}{S_{yt} + S_{yc}} \quad (2-21)$$

### 2.5. Maximum-Normal-Stress Theory for Brittle Materials.

The maximum-normal-stress (MNS) theory states that failure occurs whenever one of the three principal stresses equals or exceeds the strength. Again we arrange the principal stresses for a general stress state in the ordered form  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ . This theory then predicts that failure occurs whenever :

$$\sigma_1 \geq S_{ut} \quad \text{or} \quad \sigma_3 \leq -S_{uc} \quad (2-22)$$

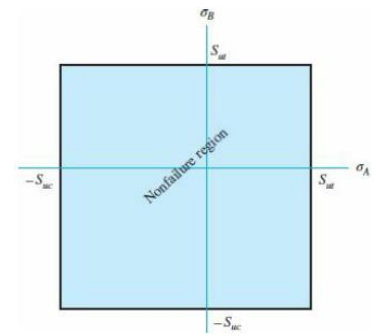
where  $S_{ut}$  and  $S_{uc}$  are the ultimate tensile and compressive strengths, respectively, given as positive quantities.

For plane stress, with the principal stresses, with  $\sigma_A \geq \sigma_B$ , Eq. (2-22) can be written as:

$$\sigma_A \geq S_{ut} \quad \text{or} \quad \sigma_B \leq -S_{uc} \quad (2-23)$$

which is plotted in Figure below. As before, the failure criteria equations can be converted to design equations. We can consider two sets of equations for load lines where  $\sigma_A \geq \sigma_B$  as :

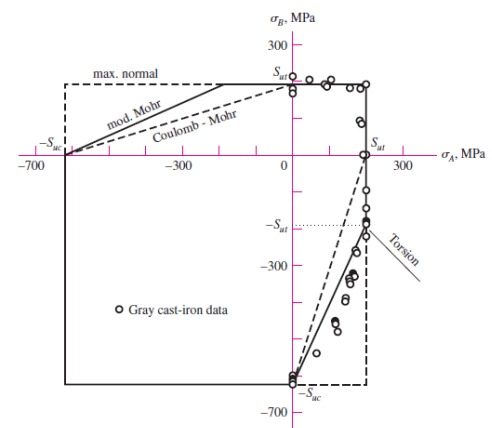
$$\sigma_A = \frac{S_{ut}}{n} \quad \text{or} \quad \sigma_B = -\frac{S_{uc}}{n} \quad (2-24)$$



### 2.6 Modifications of the Mohr Theory for Brittle Materials.

We will discuss two modifications of the Mohr theory for brittle materials: the Brittle- Coulomb-Mohr (BCM) theory and the modified Mohr (MM) theory. The equations provided for the theories will be restricted to plane stress and be of the design type incorporating the factor of safety.

The Coulomb-Mohr theory was discussed earlier in Sec. 2-4 with Eqs. (2-17) to (2-19). Written as design equations for a brittle material, they are:



#### Brittle-Coulomb-Mohr.

$$\sigma_A = \frac{S_{ut}}{n} \quad \sigma_A \geq \sigma_B \geq 0 \quad (2-25)$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \quad \sigma_A \geq 0 \geq \sigma_B \quad (2-26)$$

$$\sigma_B = -\frac{S_{uc}}{n} \quad 0 \geq \sigma_A \geq \sigma_B \quad (2-27)$$



**Modified Mohr**

$$\sigma_A = \frac{S_{ut}}{n} \quad \sigma_A \geq \sigma_B \geq 0 \quad (2-28)$$

$$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left| \frac{\sigma_B}{\sigma_A} \right| \leq 1$$

$$\frac{(S_{uc} - S_{ut}) \sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \quad \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left| \frac{\sigma_B}{\sigma_A} \right| > 1 \quad (2-29)$$

$$\sigma_B = -\frac{S_{uc}}{n} \quad 0 \geq \sigma_A \geq \sigma_B \quad (2-30)$$

Data are still outside this extended region. The straight line introduced by the modified Mohr theory, for  $\sigma_A \geq 0 \geq \sigma_B$  and  $|\sigma_B/\sigma_A| > 1$ , can be replaced by a parabolic relation which can more closely represent some of the data. However, this introduces a nonlinear equation for the sake of a minor correction, and will not be presented here.

**2.7 Introduction to Fracture Mechanics:**

The idea that cracks exist in parts even before service begins, and that cracks can grow during service, has led to the descriptive phrase “damage-tolerant design.” The focus of this philosophy is on crack growth until it becomes critical, and the part is removed from service.

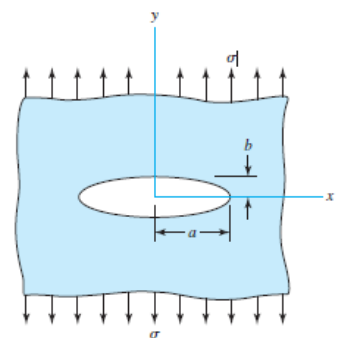
**Quasi-Static Fracture**

The foundation of fracture mechanics was first established by Griffith using the stress field calculations for an elliptical flaw in a plate. For the infinite plate loaded by an applied uniaxial stress  $\sigma$  in Figure below, the maximum stress occurs at  $(\pm a, 0)$  and is given by

$$(\sigma_y)_{\max} = (1 + 2 a/b) \sigma \quad (2-31)$$

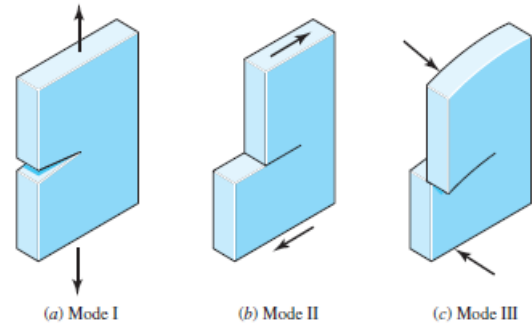
Note that when  $a = b$ , the ellipse becomes a circle and Eq. (2-31) gives a stress concentration factor of 3. This agrees with the well-known result for an infinite plate with a circular hole (see Table A-13-1). For a fine crack,  $b/a \rightarrow 0$ , and Eq. (2-30) predicts that  $(\sigma_y)_{\max} \rightarrow \infty$ .

Griffith showed that the crack growth occurs when the energy release rate from applied loading is greater than the rate of energy for crack growth. Crack growth can be stable or unstable. Unstable crack growth occurs when the rate of change of the energy release rate relative to the crack length is equal to or greater than the rate of change of the crack growth rate of energy.



### Crack Modes and the Stress Intensity Factor

Three distinct modes of crack propagation exist, as shown in Figure. A tensile stress field gives rise to mode I, the *opening crack propagation mode*, as shown in Fig. a. This mode is the most common in practice. Mode II is the *sliding mode*, is due to in-plane shear, and can be seen in Fig. b. Mode III is the *tearing mode*, which arises from out-of-plane shear, as shown in Fig. c. Combinations of these modes can also occur. Since mode I is the most common and important mode, the remainder of this section will consider only this mode.



The stress intensity factor is a function of geometry, size and shape of the crack, and the type of loading. For various load and geometric configurations, the stress intensity factor of mode I can be written as

$$K_I = \beta \sigma \sqrt{\pi a} \quad (2-32)$$

where  $\beta$  is the *stress intensity modification factor*. Tables for  $\beta$  are available in the literature for basic configurations. Figures 5–25 to 5–30 present a few examples of  $\beta$  for mode I crack propagation.

### Fracture Toughness

When the magnitude of the mode I stress intensity factor reaches a critical value,  $K_{Ic}$ , crack propagation initiates. The critical stress intensity factor  $K_{Ic}$  is a material property that depends on the material, crack mode, processing of the material, temperature, loading rate, and the state of stress at the crack site (such as plane stress versus plane strain). The critical stress intensity factor  $K_{Ic}$  is also called the fracture toughness of the material.

Notes:

- As long as the stress intensity factor  $K$  stays below a critical value called the fracture toughness,  $K_c$ , the crack is considered stable.
- If  $K$  reaches  $K_c$ , the crack will propagate and lead to sudden failure.
- Propagation rates can reach 1 mile/sec.
- Fracture toughness is a material property.

The strength-to-stress ratio  $K_{Ic}/K_I$  can be used as a factor of safety as

$$n = K_{Ic}/K_I$$

**Example:** An ASTM cast iron has minimum ultimate strengths of 30 kpsi in tension and 100 kpsi in compression. Find the factors of safety using the MNS, BCM, and MM theories for each of the following stress states. Plot the failure diagrams in the  $\sigma_A, \sigma_B$  plane to scale and locate the coordinates of each stress state.

- (a)  $\sigma_x = 20$  kpsi,  $\sigma_y = 6$  kpsi;                      (b)  $\sigma_x = 12$  kpsi,  $\tau_{xy} = -8$  kpsi  
(c)  $\sigma_x = -6$  kpsi,  $\sigma_y = -10$  kpsi,  $\tau_{xy} = -5$  kpsi;                      (d)  $\sigma_x = -12$  kpsi,  $\tau_{xy} = 8$  kpsi

Solution:

$$S_{ut} = 30 \text{ kpsi}, S_{uc} = 100 \text{ kpsi}; \sigma_A = 20 \text{ kpsi}, \sigma_B = 6 \text{ kpsi}$$

(a) MNS: 
$$n = \frac{S_{ut}}{\sigma_x} = \frac{30}{20} = 1.5 \text{ Ans.}$$

BCM: 
$$n = \frac{30}{20} = 1.5 \text{ Ans.}$$

MM: 
$$n = \frac{30}{20} = 1.5 \text{ Ans.}$$

- (b)  $\sigma_x = 12$  kpsi,  $\tau_{xy} = -8$  kpsi

$$\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16, -4 \text{ kpsi}$$

MNS: 
$$n = \frac{30}{16} = 1.88 \text{ Ans.}$$

BCM: 
$$\frac{1}{n} = \frac{16}{30} - \frac{(-4)}{100} \Rightarrow n = 1.74 \text{ Ans.}$$

MM: 
$$n = \frac{30}{16} = 1.88 \text{ Ans.}$$

- (c)  $\sigma_x = -6$  kpsi,  $\sigma_y = -10$  kpsi,  $\tau_{xy} = -5$  kpsi

$$\sigma_A, \sigma_B = \frac{-6 - 10}{2} \pm \sqrt{\left(\frac{-6 + 10}{2}\right)^2 + (-5)^2} = -2.61, -13.39 \text{ kpsi}$$

MNS: 
$$n = -\frac{100}{-13.39} = 7.47 \text{ Ans.}$$

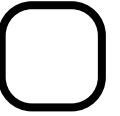
BCM: 
$$n = -\frac{100}{-13.39} = 7.47 \text{ Ans.}$$

MM: 
$$n = -\frac{100}{-13.39} = 7.47 \text{ Ans.}$$

- (d)  $\sigma_x = -12$  kpsi,  $\tau_{xy} = 8$  kpsi

$$\sigma_A, \sigma_B = -\frac{12}{2} \pm \sqrt{\left(-\frac{12}{2}\right)^2 + 8^2} = 4, -16 \text{ kpsi}$$

MNS: 
$$n = \frac{-100}{-16} = 6.25 \text{ Ans.}$$

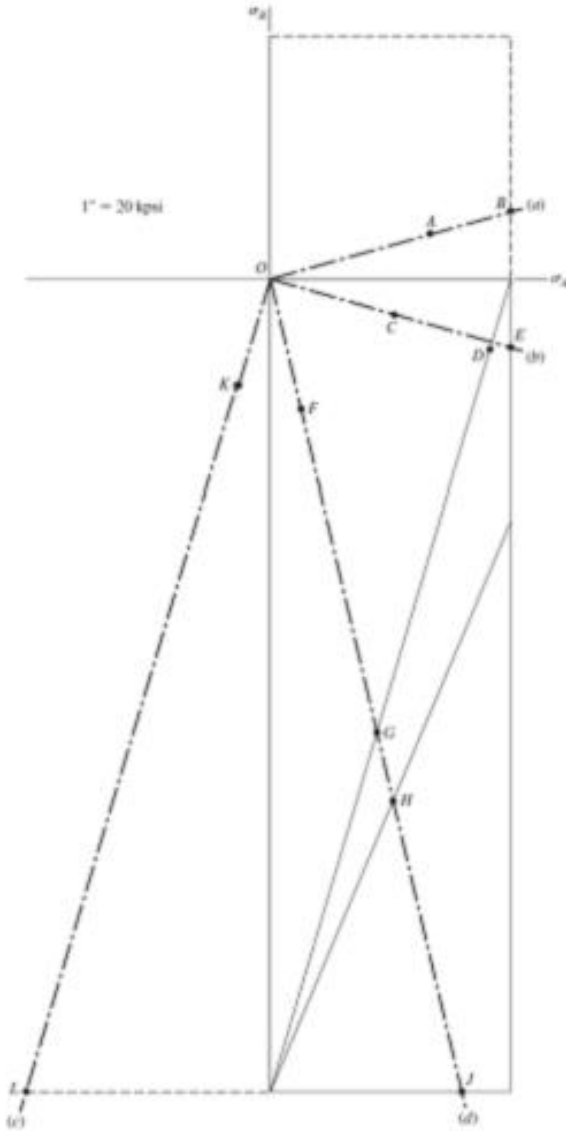


BCM:

$$\frac{1}{n} = \frac{4}{30} - \frac{(-16)}{100} \Rightarrow n = 3.41 \text{ Ans.}$$

MM:

$$\frac{1}{n} = \frac{(100 - 30)4}{100(30)} - \frac{-16}{100} \Rightarrow n = 3.95 \text{ Ans.}$$



**Example:** For example above, case (d), estimate the factors of safety from the three theories by graphical measurements of the load line.

Solution:

(a) For all methods:  $n = \frac{OB}{OA} = \frac{1.55}{1.03} = 1.5$

(b) BCM:  $n = \frac{OD}{OC} = \frac{1.4}{0.8} = 1.75$

All other methods:  $n = \frac{OE}{OC} = \frac{1.55}{0.8} = 1.9$

(c) For all methods:  $n = \frac{OL}{OK} = \frac{5.2}{0.68} = 7.6$

(d) MNS:  $n = \frac{OJ}{OF} = \frac{5.12}{0.82} = 6.2$

BCM:  $n = \frac{OG}{OF} = \frac{2.85}{0.82} = 3.5$

MM:  $n = \frac{OH}{OF} = \frac{3.3}{0.82} = 4.0$

## CHAPTER THREE

### Fatigue Failure Resulting from Variable Loading

#### *3.1 Introduction:*

We have discussed, in the previous chapter, the stresses due to static loading only. But only a few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses. Therefore the **fatigue** can be define by **it is condition where by a material cracks or fails as a result of repeated cyclic stress or progressive localized permanent structure changes.**

#### *3.2 Fatigue-Life Methods*

The three major fatigue life methods used in design and analysis are the stress-life method, the strain-life method, and the linear-elastic fracture mechanics method. These methods attempt to predict the life in number of cycles to failure,  $N$ , for a specific level of loading. Life of  $1 \leq N \leq 10^3$  cycles is generally classified as low-cycle fatigue, whereas high-cycle fatigue is considered to be  $N > 10^3$  cycles.

The stress-life method, based on stress levels only, is the least accurate approach, especially for low-cycle applications. However, it is the most traditional method, since it is the easiest to implement for a wide range of design applications, has ample supporting data, and represents high-cycle applications adequately.

The strain-life method involves more detailed analysis of the plastic deformation at localized regions where the stresses and strains are considered for life estimates. This method is especially good for low-cycle fatigue applications. In applying this method, several idealizations must be compounded, and so some uncertainties will exist in the results. For this reason, it will be discussed only because of its value in adding to the understanding of the nature of fatigue.

The fracture mechanics method assumes a crack is already present and detected. It is then employed to predict crack growth with respect to stress intensity. It is most practical when applied to large structures in conjunction with computer codes and a periodic inspection program.

### 3.3 The Stress-Life Method:

To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the cycles or stress reversals are counted to destruction. The most widely used fatigue-testing device is the R. R. Moore high-speed rotating-beam machine. This machine subjects the specimen to pure bending (no transverse shear) by means of weights. The specimen shown in Figure. (3-1) is very carefully machined and polished, with a final polishing in an axial direction to avoid circumferential scratches. Other fatigue-testing machines are available for applying fluctuating or reversed axial stresses, torsional stresses, or combined stresses to the test specimens.

To establish the fatigue strength of a material, quite a number of tests are necessary because of the statistical nature of fatigue. For the rotating-beam test, a constant bending load is applied, and the number of revolutions (stress reversals) of the beam required for failure is recorded. The first test is made at a stress that is somewhat under the ultimate strength of the material. The second test is made at a stress that is less than that used in the first. This process is continued, and the results are plotted as an S-N diagram (Fig. 3-2). This chart may be plotted on semilog paper or on log-log paper. In the case of ferrous metals and alloys, the graph becomes horizontal after the material has been stressed for a certain number of cycles. Plotting on log paper emphasizes the bend in the curve, which might not be apparent if the results were plotted by using Cartesian coordinates.

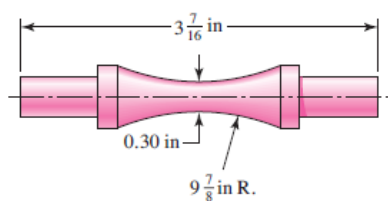


Fig.3-1

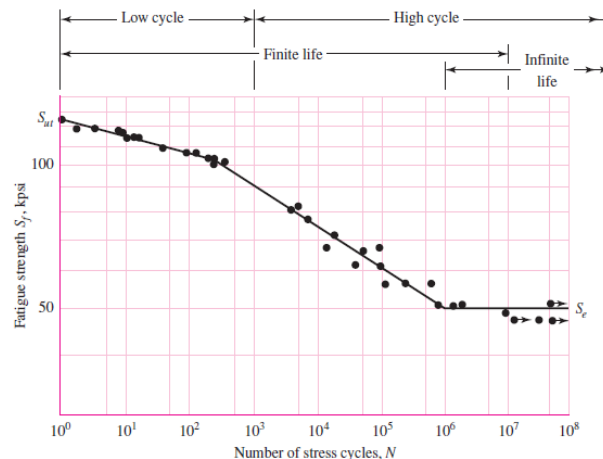


Fig.3-2

The ordinate of the S-N diagram is called the fatigue strength  $S_f$ ; a statement of this strength value must always be accompanied by a statement of the number of cycles  $N$  to which it corresponds.

Soon we shall learn that S-N diagrams can be determined either for a test specimen or for an actual mechanical element. Even when the material of the test specimen and that of the mechanical element are identical, there will be significant differences between the diagrams for the two.

In the case of the steels, a knee occurs in the graph, and beyond this knee failure will not occur, no matter how great the number of cycles. The strength corresponding to the knee is called the endurance limit  $S_e$ , or the fatigue limit. The graph of Fig. 3-2 never does become horizontal for nonferrous metals and alloys, and hence these materials do not have an endurance limit.

The S-N diagram is usually obtained by completely reversed stress cycles, in which the stress level alternates between equal magnitudes of tension and compression. We note that a stress cycle ( $N = 1$ ) constitutes a single application and removal of a load and then another application and removal of the load in the opposite direction. Thus  $N = 1/2$  means the load is applied once and then removed, which is the case with the simple tension test.

The body of knowledge available on fatigue failure from  $N = 1$  to  $N = 1000$  cycles is generally classified as **low-cycle fatigue**, as indicated in Fig. 3-2. **High-cycle fatigue**, then, is concerned with failure corresponding to stress cycles greater than  $10^3$  cycles.

We also distinguish a **finite-life** region and an **infinite-life** region in Fig. 3-2. The boundary between these regions cannot be clearly defined except for a specific material; but it lies somewhere between  $10^6$  and  $10^7$  cycles for steels, as shown in Fig. 3-2.

As stated earlier, the stress-life method is the least accurate approach especially for low-cycle applications. However, it is the most traditional method, with much published data available. It is the easiest to implement for a wide range of design applications and represents high-cycle applications adequately. For these reasons the stress-life method will be emphasized in subsequent sections of this chapter. However, care should be exercised when applying the method for low-cycle applications, as the method does not account for the true stress-strain behavior when localized yielding occurs.

### 3.4 Crack Growth:

Fatigue cracks nucleate and grow when stresses vary and there is some tension in each stress cycle. Consider the stress to be fluctuating between the limits of  $\sigma_{\min}$  and  $\sigma_{\max}$ , where the stress range is defined as  $\sigma = \sigma_{\max} - \sigma_{\min}$ . From Equation of the stress intensity is given by  $K_I = \beta\sigma\sqrt{\pi a}$ . Thus, for  $\Delta\sigma$ , the stress intensity range per cycle is:

$$\Delta K_I = \beta(\sigma_{\max} - \sigma_{\min})\sqrt{\pi a} = \beta\Delta\sigma\sqrt{\pi a} \quad 3-1$$

To develop fatigue strength data, a number of specimens of the same material are tested at various levels of  $\Delta\sigma$ . Cracks nucleate at or very near a free surface or large discontinuity. Assuming an initial crack length of  $a_i$ , crack growth as a function of the number of stress cycles  $N$  will depend on  $\sigma$ , that is,  $\Delta K_I$ . For  $K_I$  below some threshold value  $(\Delta K_I)_{th}$  a crack will not grow.



Figure 3-3 represents the crack length  $a$  as a function of  $N$  for three stress levels  $(\Delta\sigma)_3 > (\Delta\sigma)_2 > (\Delta\sigma)_1$ , where  $(\Delta K_i)_3 > (\Delta K_i)_2 > (\Delta K_i)_1$  for a given crack size. Notice the effect of the higher stress range in Fig. 3-3 in the production of longer cracks at a particular cycle. When the rate of crack growth per cycle,  $da/dN$  in Fig. 3-3, is plotted as shown in Fig. 3-4, the data from all three stress range levels superpose to give a sigmoidal curve. The three stages of crack development are observable, and the stage II data are linear on log-log coordinates, within the domain of linear elastic fracture mechanics (LEFM) validity. A group of similar curves can be generated by changing the stress ratio  $R = \sigma_{\min}/\sigma_{\max}$  of the experiment.

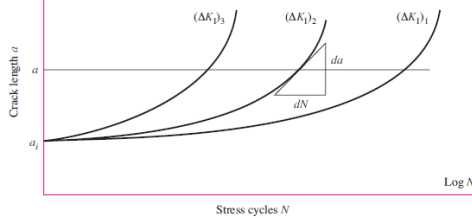


Fig. 3-3

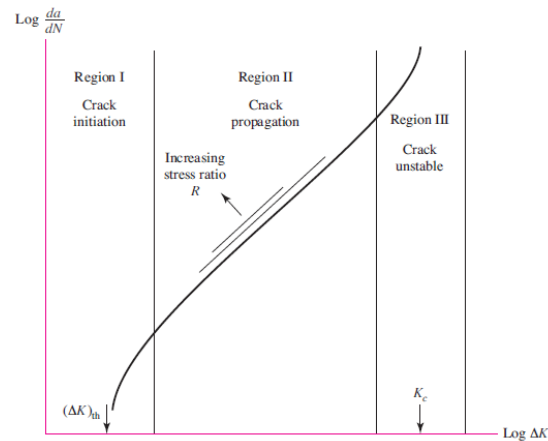


Fig. 3-4

Table 3-1 Conservative Values of Factor C and Exponent m in Eq. (3-1) for Various Forms of Steel ( $R = 0$ ).

Material	$C, \frac{m/\text{cycle}}{(\text{MPa}\sqrt{m})^m}$	$C, \frac{\text{in}/\text{cycle}}{(\text{kpsi}\sqrt{\text{in}})^m}$	$m$
Ferritic-pearlitic steels	$6.89(10^{-12})$	$3.60(10^{-10})$	3.00
Martensitic steels	$1.36(10^{-10})$	$6.60(10^{-9})$	2.25
Austenitic stainless steels	$5.61(10^{-12})$	$3.00(10^{-10})$	3.25

Here we present a simplified procedure for estimating the remaining life of a cyclically stressed part after discovery of a crack. This requires the assumption that plane strain conditions prevail. Assuming a crack is discovered early in stage II, the crack growth in region II of Fig. 3-4 can be approximated by the Paris equation, which is of the form:

$$\frac{da}{dN} = C(\Delta K_I)^m \quad 3-2$$

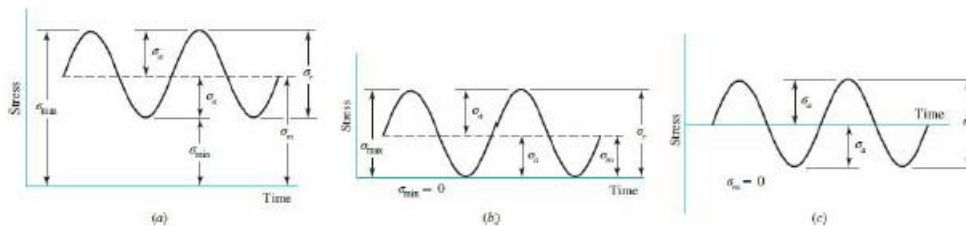
where  $C$  and  $m$  are empirical material constants and  $\Delta K_I$  is given by Eq. 3-1. Representative, but conservative, values of  $C$  and  $m$  for various classes of steels are listed in Table 3-1. Substituting Eq.3-1 and integrating gives:

$$\int_0^{N_f} dN = N_f = \frac{1}{C} \int_{a_i}^{a_f} \frac{da}{(\beta \Delta \sigma \sqrt{\pi a})^m} \quad 3-3$$

Here  $a_i$  is the initial crack length,  $a_f$  is the final crack length corresponding to failure, and  $N_f$  is the estimated number of cycles to produce a failure after the initial crack is formed. Note that  $b$  may vary in the integration variable (e.g., see Figs. 5-25 to 5-30).

### 3.5 The Stress-Life Method and the S-N Diagram

The stress-life method relies on studies of test specimens subjected to control cycling between two stress levels, known as constant amplitude loading. Figure below (a) shows a general case of constant amplitude loading between a minimum and maximum stress. Load histories of actual parts are often much more diverse, with variable amplitude loading, but many cases can be reasonably modeled with the constant amplitude approach. This is especially true for rotating equipment that experiences repetitive loading with each revolution. Testing with constant amplitude loading also provides a controlled environment to study the nature of fatigue behavior and material fatigue properties.



Constant amplitude loading. (a) General; (b) Repeated, with  $\sigma_{min} = 0$ ; (c) Completely reversed, with  $\sigma_m = 0$

If the largest force is  $F_{max}$  and the smallest force is  $F_{min}$ , then a steady component and an alternating component can be constructed as follows:

$$F_m = \frac{F_{max} + F_{min}}{2} \quad F_a = \left| \frac{F_{max} - F_{min}}{2} \right|$$

Where  $F_m$  is the midrange steady component of force, and  $F_a$  is the amplitude of the alternating component of force.

The constant amplitude stress situation is characterized by the following terminology:

$\sigma_{min}$  = minimum stress

$\sigma_{max}$  = maximum stress

$\sigma_m$  = mean stress, or midrange stress

$\sigma_a$  = alternating stress, or stress amplitude

$\sigma_r$  = stress range

$$\sigma_a = \left| \frac{\sigma_{max} - \sigma_{min}}{2} \right|$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

The following relations are evident from Figure a,

The stress ratio  $R = \frac{\sigma_{min}}{\sigma_{max}}$

The amplitude ratio  $A = \frac{\sigma_a}{\sigma_m}$

The stress ratio can have values between  $-1$  and  $+1$ , and is commonly used to represent with a single value the nature of the stress pattern. For example,  $R = -1$  is completely reversed,  $R = 0$  is repeated load,  $R = 1$  is steady.

The mean stress can have positive or negative values, while the alternating stress is always a positive magnitude representing the amplitude of the stress that alternates above and below the mean stress.

Two special cases are common enough to warrant special attention. Figure (b) shows what is called a repeated stress, in which the stress cycles between a minimum stress of zero to a maximum stress. Figure (c) shows a completely reversed stress, in which the stress alternates between equal magnitudes of tension and compression, with a mean stress of zero. We shall add the subscript r to the alternating stress, that is  $\sigma_{ar}$ , when it is advantageous to clarify that an alternating stress is completely reversed. Most fatigue testing is done with completely reversed stresses; then the modifying effect of nonzero mean stress is considered separately.

### ***3.6 Estimating the Endurance Limit:***

Now to present a method for estimating endurance limits. Note that estimates obtained from quantities of data obtained from many sources probably have a large spread and might deviate significantly from the results of actual laboratory tests of the mechanical properties of specimens obtained through strict purchase-order specifications. Since the area of uncertainty is greater, compensation must be made by employing larger design factors than would be used for static design.

For steels, simplifying our observation of Fig. 6-17, we will estimate the endurance limit as:

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \quad 3-4$$

where  $S_{ut}$  is the minimum tensile strength. The prime mark on  $S'_e$  in this equation refers to the rotating-beam specimen itself (test specimen).

The endurance limits for various classes of cast irons, polished or machined, are given in Table A-22. Aluminum alloys do not have an endurance limit. The fatigue strengths of some aluminum alloys at  $5(10^8)$  cycles of reversed stress are given in Table A-22.

### 3.7 Fatigue Strength:

#### Estimating the Fatigue Strength at $10^3$ Cycles

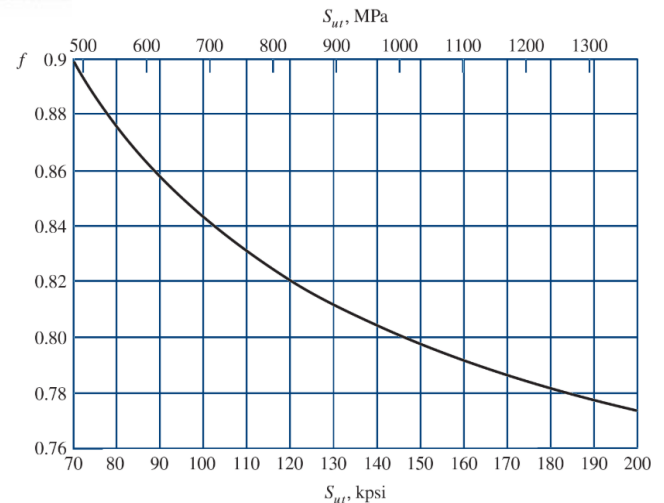
As shown in Fig. 6–10, a region of low-cycle fatigue extends from  $N = 1$  to about  $10^3$  cycles. In this region the fatigue strength  $S_f$  is only slightly smaller than the tensile strength  $S_{ut}$ .

However, it has been observed that for steels,  $f$  is generally a little lower for higher strength materials. A relationship between  $f$  and  $S_{ut}$  has been developed based on the elastic strain line in the strain-life approach to fatigue analysis. The resulting relationship for steels is plotted in Figure below and expressed by the curve-fit equations

$$f = 1.06 - 2.8(10^{-3})S_{ut} + 6.9(10^{-6})S_{ut}^2 \quad 70 < S_{ut} < 200 \text{ kpsi} \quad 3-5$$

$$f = 1.06 - 4.1(10^{-4})S_{ut} + 1.5(10^{-7})S_{ut}^2 \quad 500 < S_{ut} < 1400 \text{ MPa}$$

Values for  $f$  can be estimated from the plot or equations 3-5, noting that they are not experimentally based and are only intended to provide a better estimate than using a fixed value. Figure is a plot of  $f$  for  $490 \leq S_{ut} \leq 1400$  MPa. For values of  $S_{ut}$  lower than the limit given,  $f = 0.9$  is recommended.



Low-cycle fatigue is often defined (see Fig. 6–10) as failure that occurs in a range of  $1 \leq N \leq 10^3$  cycles. On a loglog plot such as Fig. 6–10 the failure locus in this range is nearly linear below  $10^3$  cycles. A straight line between  $10^3, f S_{ut}$  and  $1, S_{ut}$  (transformed) is conservative, and it is given by

$$S_f \geq S_{ut} N^{(\log f)/3} \quad 1 \leq N \leq 10^3$$

#### The High-Cycle S-N Line

The relationship between fatigue strength and life in the high-cycle finite-life region, which is between  $10^3$  and  $10^6$  cycles, is approximately linear on the log-log scale. It thus can be represented on a normal scale by a power function known as Basquin's equation,

$$S_f = aN^b \quad 3-6$$

where  $S_f$  is the fatigue strength correlating to a life  $N$  in cycles to failure. The constants  $a$  and  $b$  are the ordinate intercept and the slope of the line in log-log coordinates, which can be readily recognized by taking the logarithm of both sides of Equation above, giving  $\log S_f = b \log N + \log a$ .

To obtain expressions for  $a$  and  $b$ , substitute into Equation (3-6) for  $(N, S_f)$  the two known points  $(10^3, fS_{ut})$  and  $(10^6, S_e)$ . Solving for  $a$  and  $b$ ,

$$a = \frac{(fS_{ut})^2}{S_e} \quad 3-7$$

$$b = -\frac{1}{3} \log \left( \frac{fS_{ut}}{S_e} \right) \quad 3-8$$

Equation (3-6) can be solved for the life in cycles correlating to a completely reversed stress, replacing  $S_f$  with  $\sigma_{ar}$  or  $\sigma_a$ , the number of cycles-to-failure can be expressed as:

$$N = \left( \frac{\sigma_a}{a} \right)^{1/b} \quad 3-9$$

The typical S-N diagram is only applicable for completely reversed loading. For general fluctuating loading situations, the effect of mean stress must be accounted for Fluctuating Stresses. This will lead to an equivalent completely reversed stress that is considered to be equally as damaging as the actual fluctuating stress, and which can therefore be used with the S-N diagram and Equation (3-9). Basquin's equation is commonly encountered in the research literature in terms of load reversals (two reversals per cycle) in the form

$$\sigma_{ar} = \sigma'_f (2N)^b \quad 3-10$$

Where  $\sigma_{ar}$  is the alternating stress (completely reversed),  $N$  is the number of cycles,  $b$  is the fatigue strength exponent, and is the slope of the line, and  $\sigma'_f$  is the fatigue strength coefficient.

### 3.8 Endurance Limit Modifying Factors

The rotating-beam specimen used in the laboratory to determine endurance limits is prepared very carefully and tested under closely controlled conditions. It is unrealistic to expect the endurance limit of a mechanical or structural member to match the values obtained in the laboratory. Some differences include:

- **Material:** composition, basis of failure, variability
- **Manufacturing:** method, heat treatment, fretting corrosion, surface condition, stress concentration
- **Environment:** corrosion, temperature, stress state, relaxation times
- **Design:** size, shape, life, stress state, stress concentration, speed, fretting, galling

Marin identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. The question of whether to adjust the endurance limit by subtractive corrections or multiplicative corrections was resolved by extensive analytical analysis of steel. Marin's equation is written as:

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S_e'$$

Where:  $k_a$  = surface condition modification factor

$k_b$  = size modification factor

$k_c$  = load modification factor

$k_d$  = temperature modification factor

$k_e$  = reliability factor.

$k_f$  = miscellaneous-effects modification factor

$S_e'$  = rotary-beam test specimen endurance limit

$S_e$  = endurance limit at the critical location of a machine part in the geometry and condition of use.

When endurance tests of parts are not available, estimations are made by applying Marin factors to the endurance limit. This will effectively lower the high-cycle end of the S-N line, while not moving the low-cycle end of the line. Accordingly, the modifying effects are applied proportionately through the finite life region. This seems reasonable and consistent with limited experimental data.

- **Surface Factor  $k_a$  :**

The surface modification factor depends on the quality of the finish of the actual part surface and on the tensile strength of the part material.

$$k_a = a S_{ut}^b \quad 3-12$$

Where  $S_{ut}$  is the minimum tensile strength and a and b are to be found in Table 6-2.

- **Size Factor  $k_b$ :**

The size factor has been evaluated for bending and torsion of round rotating beam may be given by:

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases} \quad 3-13$$

For axial loading there is no size effect, so:  $k_b = 1$  3-14

For non-rotating solid or hollow rounds:  $d_e = 0.370d$  3-15

A rectangular section of dimensions  $h \times b$  has:  $d_e = 0.808(hb)^{1/2}$  3-16

Where:  $d$  = actual diameter

$d_e$  = effective diameter.

- **Loading Factor  $k_c$ :**

When fatigue tests are carried out with rotating bending, axial (push-pull), and torsional loading, the endurance limits differ with  $S_{ut}$ . Here, we will specify average values of the load factor as:

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \quad 3-17$$

When torsion is combined with other loading, such as bending, set  $k_c = 1$ .

- **Temperature Factor  $k_d$  :**

When operating temperatures are below room temperature, brittle fracture is a strong possibility and should be investigated first. When the operating temperatures are higher than room temperature, yielding should be investigated first because the yield strength drops off so rapidly with temperature. Any stress will induce creep in a material operating at high temperatures; so this factor must be considered too.

Temperature, °C	$S_T/S_{RT}$	Temperature, °F	$S_T/S_{RT}$
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

If the temperature-specific ultimate strength is not available, the curve-fit polynomials can be utilized (for steels) represented by

$$\begin{aligned} S_T/S_{RT} &= 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2 \\ S_T/S_{RT} &= 0.99 + 5.9(10^{-4})T_C - 2.1(10^{-6})T_C^2 \end{aligned} \quad 3-18$$

Where  $S_T$  and  $S_{RT}$  are the ultimate strengths at the operating temperature and room temperature, respectively, and  $T_F$  and  $T_C$  is the operating temperature in degrees Fahrenheit and Celsius, respectively. Equation (3-18) is for steels, and should be limited to the range 20°C (70°F) to 550°C (1000°F).

In the case where the endurance limit is known (e.g., by testing, or tabulated material data) at room temperature, it can be adjusted for the operating temperature by applying

$$k_d = \frac{S_T}{S_{RT}} \quad 3-19$$

Again, though, if the endurance limit is available or being estimated based on the ultimate strength at the operating temperature, it needs no further adjustment and  $k_d$  should be set to unity.



- **Reliability Factor  $k_e$ :**

The reliability factor account only for the scatter in the endurance limit fatigue data and is not part of a complete stochastic analysis. The reliability modification factor to account for this can be written as:

$$k_e = 1 - 0.08 z_a \quad 3-20$$

Where  $z_a$  values for any desired reliability can be determined from Table A-10. Table 6-5 gives reliability factors for some standard specified reliability.

- **Miscellaneous-Effects Factor  $k_f$ :**

Though the factor  $k_f$  is intended to account for the reduction in endurance limit due to all other effects, it is really intended as a reminder that these must be accounted for, because actual values of  $k_f$  are not always available.

### 3.9 Stress Concentration and Notch Sensitivity:

Previously, it was pointed out that the existence of irregularities or discontinuities, such as holes, grooves, or notches, in a part increases the theoretical stresses significantly in the immediate vicinity of the discontinuity. Equation below defined a stress-concentration factor  $K_t$  (or  $K_{ts}$ ), which is used with the nominal stress to obtain the maximum resulting stress very local to the irregularity or defect.

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \quad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

The theoretical stress-concentration factor  $K_t$  is defined for static loading conditions. Under dynamic loading conditions leading to fatigue, it turns out that the fatigue strength of a notched specimen is often not affected as much as would be expected from applying the stress-concentrated maximum stress. Consequently, for fatigue purposes, a *fatigue stress-concentration factor*,  $K_f$  (also known as the fatigue notch factor), is defined based on the fatigue strengths of notch-free versus notched specimens at long-life conditions (e.g., the endurance limit at  $10^6$  cycles) under completely reversed loading, such that

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}} \quad 3-21$$

The maximum stress is, in fact,  $\sigma_{\max} = K_f \sigma_0$  or  $\tau_{\max} = K_{fs} \tau_0$  3-22

Where  $K_f$  is a reduced value of  $K_t$  and  $\sigma_0$  is the nominal stress. The factor  $K_f$  is commonly called a fatigue stress-concentration factor, and hence the subscript f. So it is convenient to think of  $K_f$  as a stress-concentration factor reduced from  $K_t$  because of lessened sensitivity to notches.



Notch sensitivity  $q$  is defined by the equation:

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1} \quad 3-23$$

where  $q$  is usually between zero and unity. Equation (3-23) shows that if  $q = 0$ , then  $K_f = 1$ , and the material has no sensitivity to notches at all. On the other hand, if  $q = 1$ , then  $K_f = K_t$ , and the material has full notch sensitivity. In analysis or design work, find  $K_t$  first, from the geometry of the part. Then specify the material, find  $q$ , and solve for  $K_f$  from the equation

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1) \quad 3-24$$

For steels and 2024 aluminum alloys, use Fig. 6-20 to find  $q$  for bending and axial loading. For shear loading, use Fig. 6-21. In using these charts it is well to know that the actual test results from which the curves were derived exhibit a large amount of scatter. Because of this scatter it is always safe to use  $K_f = K_t$  if there is any doubt about the true value of  $q$ . Also, note that  $q$  is not far from unity for large notch radii.

The notch sensitivity of the **cast irons** is very low, varying from 0 to about 0.2, depending upon the tensile strength. To be on the conservative side, it is recommended that the value  $q = 0.2$  be used for all grades of cast iron.

Figure 6-20 has as its basis the **Neuber equation**, which is given by:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \quad 3-25$$

Where  $\sqrt{a}$  is defined as the **Neuber constant** and is a material constant. Eqs. (3-23) and (3-25) yields the notch sensitivity equation:

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \quad 3-26$$

For steel, with  $S_{ut}$  in kpsi, the Neuber constant can be approximated by a third-order polynomial fit of data as or given in Table 6-15:

Bending or axial:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad 50 \leq S_{ut} \leq 250 \text{ kpsi}$$

$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.60(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \quad 340 \leq S_{ut} \leq 1700 \text{ MPa}$$

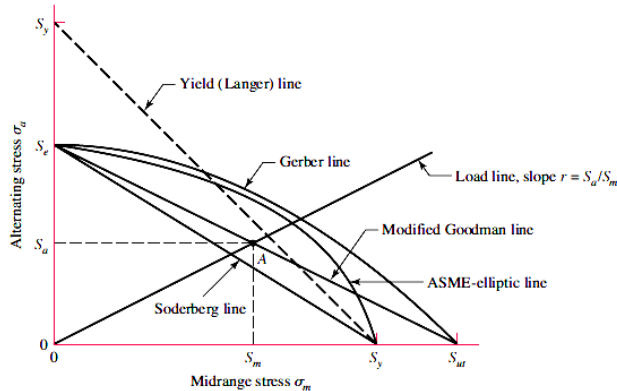
Torsion:

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad 50 \leq S_{ut} \leq 220 \text{ kpsi}$$

$$\sqrt{a} = 0.958 - 1.83(10^{-3})S_{ut} + 1.43(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \quad 340 \leq S_{ut} \leq 1500 \text{ MPa}$$

3-27

### 3.10 Fatigue Failure Criteria for Fluctuating Stress:



Five criteria of failure are diagrammed in Figure above, the Soderberg, the modified Goodman, the Gerber, the ASME-elliptic, and yielding. The diagram shows that only the Soderberg criterion guards against any yielding, but is biased low. Considering the modified Goodman line as a criterion, point A represents a limiting point with an alternating strength  $S_a$  and midrange strength  $S_m$ . The slope of the load line shown is defined as  $r = S_a/S_m$ .

The Goodman line connects two points: the endurance limit when the mean stress is zero (completely reversed) and the ultimate strength when the alternating stress is zero (steady stress).

#### Goodman

The Goodman line is simple, conservative, and good for design purposes. It is almost entirely to the conservative side of the data, and therefore not good when seeking a criterion that is typical of the data. It is only applicable for positive mean stress, as it is non-conservative if applied to negative mean stress.

*Failure criterion:*

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \quad 3-28$$

*Design equation:* The stresses  $\sigma_{sa}$  and  $\sigma_{sm}$  can replace  $S_a$  and  $S_m$ , where  $n$  is the design factor or factor of safety.

$$n_f = \left( \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} \quad \sigma_m \geq 0 \quad 3-29$$

#### Gerber

The Gerber curve is a parabolic equation that was one of the early options presented to better pass through the middle of the fatigue points. It certainly fits the data better than the Goodman line, but it can be slightly non-conservative, especially for stress conditions near the ordinate axis. It only applies to positive mean stress, as it is entirely too conservative if applied to negative mean stress. It is historically well known as a fit to the data, but there are other curves that fit better.

*Failure criterion:*

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \quad 3-30$$

*Design equation:*

$$n_f = \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m}\right)^2 \left(\frac{\sigma_a}{S_e}\right) \left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \quad \sigma_m \geq 0 \quad 3-31$$

### Soderberg

The Soderberg line is historically well known. It simply replaces the ultimate strength in the Goodman criterion with the yield strength. This makes the line ultra-conservative. Its sole purpose is to provide a simple, conservative line that checks for infinite-life fatigue and yielding at the same time. It removes the need for a separate yield check. It is perhaps useful for quick first estimates or for situations that can be grossly over-designed.

*Failure criterion:*

$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1 \quad 3-32$$

*Design equation:*

$$n = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y}\right)^{-1} \quad \sigma_m \geq 0 \quad 3-33$$

### ASME-elliptic

This criterion uses an elliptic equation to attempt to mix the qualities of the Gerber and the Soderberg criteria, that is, to fit the fatigue data, but check for yielding at the same time. As might be expected, it does both tasks to some extent, but neither very well. It is sometimes conservative and sometimes non-conservative, for both fatigue and yielding.

*Failure criterion:*

$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1 \quad 3-34$$

*Design equation:*

$$n_f = \left[\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2\right]^{-1/2} \quad \sigma_m \geq 0 \quad 3-35$$

### Langer first-cycle-yielding

The Langer first-cycle-yielding criterion is used in connection with the fatigue curve:

*Failure criterion:*

$$S_a + S_m = S_y \quad 3-36$$

*Design equation:*

$$\text{Langer static yield} \quad \sigma_a + \sigma_m = \frac{S_y}{n} \quad 3-37$$

The failure criteria are used in conjunction with a load line,  $r = S_a/S_m = \sigma_a/\sigma_m$ . Principal intersections are tabulated in Tables 6-6 to 6-8. Formal expressions for fatigue factor of safety are given in the lower panel of Tables 6-6 to 6-8. The first row of each table corresponds to the fatigue criterion, the second row is the static Langer criterion, and the third row corresponds to the intersection of the static and fatigue criteria. The first column gives the intersecting equations and the second column the intersection coordinates.

### 3.11 Fatigue Failure Criterion for Brittle Materials:

For many brittle materials, the first quadrant fatigue failure criterion follows a concave upward Smith-Dolan locus represented by

$$\frac{S_a}{S_e} = \frac{1 - S_m/S_{ut}}{1 + S_m/S_{ut}} \quad 3-39$$

Or as a design equation,

$$\frac{n\sigma_a}{S_e} = \frac{1 - n\sigma_m/S_{ut}}{1 + n\sigma_m/S_{ut}} \quad 3-40$$

For a radial load line of slope  $r$ , we substitute  $S_a/r$  for  $S_m$  in Equation (3-39) and solve for  $S_a$ , obtaining the intersect

$$S_a = \frac{rS_{ut} + S_e}{2} \left[ -1 + \sqrt{1 + \frac{4rS_{ut}S_e}{(rS_{ut} + S_e)^2}} \right] \quad 3-41$$

The fatigue diagram for a brittle material differs markedly from that of a ductile material because:

- Yielding is not involved since the material may not have yield strength.
- Characteristically, the compressive ultimate strength exceeds the ultimate tensile strength severalfold.
- First-quadrant fatigue failure locus is concave-upward (Smith-Dolan), for example, and as flat as Goodman. Brittle materials are more sensitive to midrange stress, being lowered, but compressive midrange stresses are beneficial.
- Not enough work has been done on brittle fatigue to discover insightful generalities, so we stay in the first and a bit of the second quadrant.

The most likely domain of designer use is in the range from  $-S_{ut} \leq \sigma_m \leq S_{ut}$ . The locus in the first quadrant is Goodman, Smith-Dolan, or something in between. The portion of the second quadrant that is used is represented by a straight line between the point's  $-S_{ut}, S_{ut}$  and  $0, S_e$ , which has the equation

$$S_a = S_e + \left( \frac{S_e}{S_{ut}} - 1 \right) S_m \quad -S_{ut} \leq S_m \leq 0 \quad (\text{for cast iron}) \quad 3-42$$

Table A-22 gives properties of gray cast iron. The endurance limit stated is really  $k_a k_b S'_e$  and only corrections  $k_c, k_d,$  and  $k_e$  need be made. The average  $k_c$  for axial and torsional loading is 0.9.

### 3.12 Torsional Fatigue Strength under Fluctuating Stresses:

For the case when the fluctuating stresses are entirely shear stresses, the fluctuating stress diagram and most of the fatigue failure criteria can be adapted by using shear stresses and shear strengths. Specifically, make the following adjustments:

- Replace normal stresses  $\sigma_m$  and  $\sigma_a$  with shear stresses  $\tau_m$  and  $\tau_a$ .
- Apply the load factor  $k_c = 0.59$  to the endurance limit.
- Replace  $S_y$  with  $S_{sy} = 0.577S_y$ , based on the relationship predicted by the distortion energy theory.
- Replace  $S_{ut}$  with  $S_{su}$ .

For most materials,  $S_{su}$  ranges from 65 to 80 percent of the ultimate strength. Lacking specific information justifying a higher value, use the conservative estimate of

$$S_{su} = 0.67S_{ut} \quad 3-38$$

This is consistent with testing on torsional strengths of common spring materials.

Alternatively, the fluctuating-stress diagram and the fatigue failure criterion can be used directly by converting the shear stresses to von Mises stresses.

### 3.13 Combinations of Loading Modes:

It may be helpful to think of fatigue problems as being in three categories:

- Completely reversing simple loads
- Fluctuating simple loads
- Combinations of loading modes

The simplest category is that of a completely reversed single stress which is handled with the S-N diagram, relating the alternating stress to a life. Only one type of loading is allowed here, and the midrange stress must be zero. The next category incorporates general fluctuating loads, using a criterion to relate midrange and alternating stresses (modified Goodman, Gerber, ASME-elliptic, or Soderberg).

Again, only one type of loading is allowed at a time. The third category, which we will develop in this section, involves cases where there are combinations of different types of loading, such as combined bending, torsion, and axial.

Previously, we learned that a load factor  $k_c$  is used to obtain the endurance limit, and hence the result is dependent on whether the loading is axial, bending, or torsion. In this section we want to answer the question, “How do we proceed when the loading is a mixture of, say, axial, bending, and torsional loads?” This type of loading introduces a few complications in that there may now exist combined normal and shear stresses, each with alternating and midrange values, and several of the factors used in determining the endurance limit depend on the type of loading. There may also be multiple stress-concentration factors, one for each mode of loading. The problem of how to deal with combined stresses was encountered when developing static failure theories. The distortion energy failure theory proved to be a satisfactory method of combining the multiple stresses on a stress element into a single equivalent von Mises stress. The same approach will be used here.

The first step is to generate two stress elements—one for the alternating stresses and one for the midrange stresses. Apply the appropriate fatigue stress concentration factors to each of the stresses; i.e., apply  $(K_f)_{\text{bending}}$  for the bending stresses,  $(K_{fs})_{\text{torsion}}$  for the torsional stresses, and  $(K_f)_{\text{axial}}$  for the axial stresses. Next, calculate an equivalent von Mises stress for each of these two stress elements,  $\sigma'_a$  and  $\sigma'_m$ . Finally, select a fatigue failure criterion (modified Goodman, Gerber, ASME-elliptic, or Soderberg) to complete the fatigue analysis. For the endurance limit,  $S_e$ , use the endurance limit modifiers,  $k_a$ ,  $k_b$ , and  $k_c$ , for bending. The torsional load factor,  $k_c = 0.59$  should not be applied as it is already accounted for in the von Mises stress calculation. The load factor for the axial load can be accounted for by dividing the alternating axial stress by the axial load factor of 0.85. However, a simpler approach is to simply use the load factor of 1 unless the axial stress is the dominant stress, in which case use a load factor of 0.85.

Consider the common case of a shaft with bending stresses, torsional shear stresses, and axial stresses. For this case, the von Mises stress is of the form  $\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2}$ . Considering that the bending, torsional, and axial stresses have alternating and midrange components, the von Mises stresses for the two stress elements can be written as

$$\sigma'_a = \left\{ \left[ (K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}} \frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3 \left[ (K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}} \right]^2 \right\}^{1/2}$$

$$\sigma'_m = \left\{ \left[ (K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}} \right]^2 + 3 \left[ (K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}} \right]^2 \right\}^{1/2}$$

For first-cycle localized yielding, the maximum von Mises stress is calculated. This would be done by first adding the axial and bending alternating and midrange stresses to obtain  $\sigma_{\max}$  and adding the alternating and midrange shear stresses to obtain  $\tau_{\max}$ . Then substitute  $\sigma_{\max}$  and  $\tau_{\max}$  into the equation for the von Mises stress. A simpler and more conservative method is to add Equations above.

That is, let

$$\sigma'_{\max} = \sigma'_a + \sigma'_m$$

If the stress components are not in phase but have the same frequency, the maxima can be found by expressing each component in trigonometric terms, using phase angles, and then finding the sum. If two or more stress components have differing frequencies, the problem is difficult; one solution is to assume that the two (or more) components often reach an in-phase condition, so that their magnitudes are additive.

## CHAPTER FOUR

# SHAFT DESIGN

A *shaft* is a rotating member, usually of circular cross section, used to transmit power or motion. It provides the axis of rotation, or oscillation, of elements such as gears, pulleys, flywheels, cranks, sprockets, and the like and controls the geometry of their motion. An *axle* is a nonrotating member that carries no torque and is used to support rotating wheels, pulleys, and the like.

In deciding on an approach to shaft sizing, it is necessary to realize that a stress analysis at a specific point on a shaft can be made using only the shaft geometry in the vicinity of that point. Thus the geometry of the entire shaft is not needed. In design it is usually possible to locate the critical areas, size these to meet the strength requirements, and then size the rest of the shaft to meet the requirements of the shaft-supported elements.

The deflection and slope analyses cannot be made until the geometry of the entire shaft has been defined. Thus deflection is a function of the geometry *everywhere*, whereas the stress at a section of interest is a function of *local geometry*. For this reason, shaft design allows a consideration of stress first. Then, after tentative values for the shaft dimensions have been established, the determination of the deflections and slopes can be made.

### 4.1 Shaft Materials:

Deflection is not affected by strength, but rather by stiffness as represented by the modulus of elasticity, which is essentially constant for all steels. For that reason, rigidity cannot be controlled by material decisions, but only by geometric decisions. Properties of the shaft locally depend on its history—cold work, cold forming, rolling of fillet features, heat treatment, including quenching medium, agitation, and tempering. Stainless steel may be appropriate for some environments.

### 4.2 Shaft layout:

The general layout of a shaft to accommodate shaft elements, e.g., gears, bearings, and pulleys, must be specified early in the design process in order to perform a free body force analysis and to obtain shear-moment diagrams. The geometry of a shaft is generally that of a stepped cylinder. The use of shaft shoulders is an excellent means of axially locating the shaft elements and to carry any thrust loads. Figure 7–1 shows an example of a stepped shaft supporting the gear of a worm-gear speed reducer. Each shoulder in the shaft serves a specific purpose, which you should attempt to determine by observation.



### 4.3 Shaft Design for Stress:

#### Critical Locations

It is not necessary to evaluate the stresses in a shaft at every point; a few potentially critical locations will suffice. Critical locations will usually be on the outer surface, at axial locations where the bending moment is large, where the torque is present, and where stress concentrations exist. By direct comparison of various points along the shaft, a few critical locations can be identified upon which to base the design. An assessment of typical stress situations will help.

Most shafts will transmit torque through a portion of the shaft. Typically the torque comes into the shaft at one gear and leaves the shaft at another gear. A free body diagram of the shaft will allow the torque at any section to be determined. The torque is often relatively constant at steady state operation. The shear stress due to the torsion will be greatest on outer surfaces.

The bending moments on a shaft can be determined by shear and bending moment diagrams. Since most shaft problems incorporate gears or pulleys that introduce forces in two planes, the shear and bending moment diagrams will generally be needed in two planes. Resultant moments are obtained by summing moments as vectors at points of interest along the shaft. The phase angle of the moments is not important since the shaft rotates. A steady bending moment will produce a completely reversed moment on a rotating shaft, as a specific stress element will alternate from compression to tension in every revolution of the shaft. The normal stress due to bending moments will be greatest on the outer surfaces. In situations where a bearing is located at the end of the shaft, stresses near the bearing are often not critical since the bending moment is small.

Axial stresses on shafts due to the axial components transmitted through helical gears or tapered roller bearings will almost always be negligibly small compared to the bending moment stress. They are often also constant, so they contribute little to fatigue. Consequently, it is usually acceptable to neglect the axial stresses induced by the gears and bearings when bending is present in a shaft. If an axial load is applied to the shaft in some other way, it is not safe to assume it is negligible without checking magnitudes.

#### Shaft Stresses

Bending, torsion, and axial stresses may be present in both midrange and alternating components. For analysis, it is simple enough to combine the different types of stresses into alternating and midrange von Mises stresses. It is sometimes convenient to customize the equations specifically for shaft applications. Axial loads are usually comparatively very small at critical locations where bending and torsion dominate, so they will be left out of the following equations. The fluctuating stresses due to bending and torsion are given by

$$\sigma_a = K_f \frac{M_a c}{I} \quad \sigma_m = K_f \frac{M_m c}{I} \quad 4-1$$

$$\tau_a = K_{fs} \frac{T_a c}{J} \quad \tau_m = K_{fs} \frac{T_m c}{J} \quad 4-2$$

Where  $M_m$  and  $M_a$  are the midrange and alternating bending moments,  $T_m$  and  $T_a$  are the midrange and alternating torques, and  $K_f$  and  $K_{fs}$  are the fatigue stress concentration factors for bending and torsion, respectively.

Assuming a solid shaft with round cross section, appropriate geometry terms can be introduced for  $c$ ,  $I$ , and  $J$  resulting in

$$\sigma_a = K_f \frac{32M_a}{\pi d^3} \quad \sigma_m = K_f \frac{32M_m}{\pi d^3} \quad 4-3$$

$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \quad \tau_m = K_{fs} \frac{16T_m}{\pi d^3} \quad 4-4$$

Using the distortion energy failure theory, the von Mises stresses with  $\sigma_x = \sigma$ , the bending stress,  $\sigma_y = 0$ , and  $\tau_{xy} = \tau$ , the torsional shear stress. Thus, for rotating round, solid shafts, neglecting axial loads, the fluctuating von Mises stresses are given by

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[ \left( \frac{32K_f M_a}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_a}{\pi d^3} \right)^2 \right]^{1/2} \quad 4-5$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[ \left( \frac{32K_f M_m}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2} \quad 4-6$$

Note that the stress concentration factors are sometimes considered optional for the midrange components with ductile materials, because of the capacity of the ductile material to yield locally at the discontinuity.

These equivalent alternating and midrange stresses can be evaluated using an appropriate failure curve on the modified Goodman diagram. For example, the fatigue failure criterion for the modified Goodman line as expressed previously is:

$$\frac{1}{n} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$

Substitution of  $\sigma'_a$  and  $\sigma'_m$  from Eqs. (4-5) and (4-6) results in

$$\frac{1}{n} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\}$$

For design purposes, it is also desirable to solve the equation for the diameter. This result in

$$d = \left( \frac{16n}{\pi} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \right)^{1/3}$$

Similar expressions can be obtained for any of the common failure criteria by substituting the von Mises stresses from Eqs. (4–5) and (4–6) into any of the failure criteria expressed. The resulting equations for several of the commonly used failure curves are summarized below. The names given to each set of equations identifies the significant failure theory, followed by a fatigue failure locus name. For example, DE-Gerber indicates the stresses are combined using the distortion energy (DE) theory, and the Gerber criterion is used for the fatigue failure.

**Where**

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

**DE-Goodman**

$$n = \frac{\pi d^3}{16} \left( \frac{A}{S_e} + \frac{B}{S_{ut}} \right)^{-1} \quad 4-7$$

$$d = \left[ \frac{16n}{\pi} \left( \frac{A}{S_e} + \frac{B}{S_{ut}} \right) \right]^{1/3} \quad 4-8$$

**DE-Gerber**

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e} \left\{ 1 + \left[ 1 + \left( \frac{2BS_e}{AS_{ut}} \right)^2 \right]^{1/2} \right\} \quad 4-9$$

$$d = \left( \frac{8nA}{\pi S_e} \left\{ 1 + \left[ 1 + \left( \frac{2BS_e}{AS_{ut}} \right)^2 \right]^{1/2} \right\} \right)^{1/3} \quad 4-10$$

**DE-ASME Elliptic**

$$\frac{1}{n} = \frac{16}{\pi d^3} \left[ 4 \left( \frac{K_f M_a}{S_e} \right)^2 + 3 \left( \frac{K_{fs} T_a}{S_e} \right)^2 + 4 \left( \frac{K_f M_m}{S_y} \right)^2 + 3 \left( \frac{K_{fs} T_m}{S_y} \right)^2 \right]^{1/2} \quad 4-11$$

$$d = \left\{ \frac{16n}{\pi} \left[ 4 \left( \frac{K_f M_a}{S_e} \right)^2 + 3 \left( \frac{K_{fs} T_a}{S_e} \right)^2 + 4 \left( \frac{K_f M_m}{S_y} \right)^2 + 3 \left( \frac{K_{fs} T_m}{S_y} \right)^2 \right]^{1/2} \right\}^{1/3} \quad 4-12$$

**DE-Soderberg**

$$\frac{1}{n} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{yt}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \quad 4-13$$

$$d = \left( \frac{16n}{\pi} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{yt}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \right)^{1/3} \quad 4-14$$

For a rotating shaft with constant bending and torsion, the bending stress is completely reversed and the torsion is steady. Equations (4–7) through (4–14) can be simplified by setting  $M_m$  and  $T_a$  equal to 0, which simply drops out some of the terms.

Note that in an analysis situation in which the diameter is known and the factor of safety is desired, as an alternative to use the specialized equations above, it is always still valid to calculate the alternating and mid-range stresses using Eqs. (4–5) and (4–6), and substitute them into one of the equations for the failure criteria, and solve directly for  $n$ . In a design situation, however, having the equations pre-solved for diameter is quite helpful.

It is always necessary to consider the possibility of static failure in the first load cycle. The Soderberg criteria inherently guards against yielding, as can be seen by noting that its failure curve is conservatively within the yield (Langer) line. The ASME Elliptic also takes yielding into account, but is not entirely conservative throughout its entire range. This is evident by noting that it crosses the yield line. The Gerber and modified Goodman criteria do not guard against yielding, requiring a separate check for yielding. A von Mises maximum stress is calculated for this purpose.

$$\begin{aligned}\sigma'_{\max} &= [(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2]^{1/2} \\ &= \left[ \left( \frac{32K_f(M_m + M_a)}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs}(T_m + T_a)}{\pi d^3} \right)^2 \right]^{1/2}\end{aligned}\quad 4-15$$

To check for yielding, this von Mises maximum stress is compared to the yield strength, as usual.

$$n_y = \frac{S_y}{\sigma'_{\max}} \quad 4-16$$

For a quick, conservative check, an estimate for  $\sigma'_{\max}$  can be obtained by simply adding  $\sigma'_a$  and  $\sigma'_m$ . ( $\sigma'_a + \sigma'_m$ ) will always be greater than or equal to  $\sigma'_{\max}$ , and will therefore be conservative.

#### 4-4 Estimating Stress Concentrations:

The stress analysis process for fatigue is highly dependent on stress concentrations. Stress concentrations for shoulders and keyways are dependent on size specifications that are not known the first time through the process. Fortunately, since these elements are usually of standard proportions, it is possible to estimate the stress-concentration factors for initial design of the shaft. These stress concentrations will be fine-tuned in successive iterations, once the details are known.

Table 7–1 summarizes some typical stress concentration factors for the first iteration in the design of a shaft. Similar estimates can be made for other features. The point is to notice that stress concentrations are essentially normalized so that they are dependent on ratios of geometry features, not on the specific dimensions. Consequently, by estimating the appropriate ratios, the first iteration values for stress concentrations can be obtained. These values can be used for initial design, then actual values inserted once diameters have been determined.

## CHAPTER FIVE

### Screws, Fasteners, and the Design of Nonpermanent Joints

The helical-thread screw was undoubtedly an extremely important mechanical invention. It is the basis of power screws, which change angular motion to linear motion to transmit power or to develop large forces (presses, jacks, etc.), and threaded fasteners, an important element in nonpermanent joints.

This book presupposes knowledge of the elementary methods of fastening. Typical methods of fastening or joining parts use such devices as bolts, nuts, cap screws, setscrews, rivets, spring retainers, locking devices, pins, keys, welds, and adhesives. Studies in engineering graphics and in metal processes often include instruction on various joining methods, and the curiosity of any person interested in mechanical engineering naturally results in the acquisition of good background knowledge of fastening methods. Contrary to first impressions, the subject is one of the most interesting in the entire field of mechanical design.

#### 5.1 Thread Standards and Definitions:

The terminology of screw threads, illustrated in Figure below, is explained as follows:

**Pitch** is the distance between adjacent thread forms measured parallel to the thread axis.

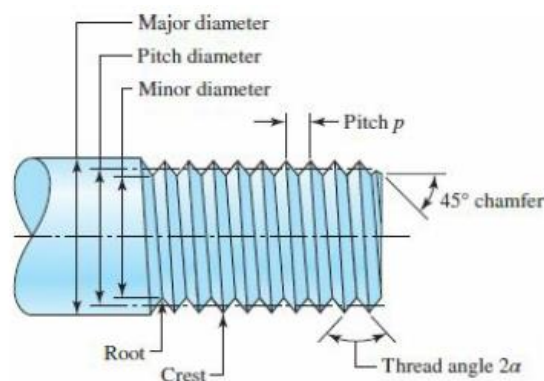
The pitch in U.S. units is the reciprocal of the number of thread forms per inch  $N$ .

**Major diameter  $d$**  is the largest diameter of a screw thread.

**Minor (or root) diameter  $d_r$**  is the smallest diameter of a screw thread.

**Pitch diameter  $d_p$**  is a theoretical diameter between the major and minor diameters.

**Lead  $l$** , not shown, is the distance the nut moves parallel to the screw axis when the nut is given one turn. For a single thread, as in Figure, the lead is the same as the pitch  $l=p$ .



A multiple-threaded product is one having two or more threads cut beside each other. Standardized products such as screws, bolts, and nuts all have single threads; a *double-threaded* screw has a lead equal to twice the pitch, a *triple-threaded* screw has a lead equal to 3 times the pitch, and so on.

All threads are made according to the right-hand rule unless otherwise noted. That is, if the bolt is turned clockwise, the bolt advances toward the nut.

Tables 8–1 and 8–2 will be useful in specifying and designing threaded parts. Note that the thread size is specified by giving the pitch  $p$  for metric sizes and by giving the number of threads per inch  $N$  for the Unified sizes.

A great many tensile tests of threaded rods have shown that an unthreaded rod having a diameter equal to the mean of the pitch diameter and minor diameter will have the same tensile strength as the threaded rod. The area of this unthreaded rod is called the tensile-stress area  $A_t$  of the threaded rod; values of  $A_t$  are listed in both tables.

Metric threads are specified by writing the diameter and pitch in millimeters, in that order. Thus, M12 \* 1.75 is a thread having a nominal major diameter of 12 mm and a pitch of 1.75 mm. Note that the letter M, which precedes the diameter, is the clue to the metric designation.

Square and Acme threads, whose profiles are shown in Fig. 8–3a and b, respectively, are used on screws when power is to be transmitted. Table 8–3 lists the preferred pitches for inch-series Acme threads. However, other pitches can be and often are used, since the need for a standard for such threads is not great.

Modifications are frequently made to both Acme and square threads. For instance, the square thread is sometimes modified by cutting the space between the teeth so as to have an included thread angle of  $10^\circ$  to  $15^\circ$ . This is not difficult, since these threads are usually cut with a single-point tool anyhow; the modification retains most of the high efficiency inherent in square threads and makes the cutting simpler. Acme threads are sometimes modified to a stub form by making the teeth shorter. This results in a larger minor diameter and a somewhat stronger screw.

### 5.2 The Mechanics of Power Screws:

A power screw is a device used in machinery to change angular motion into linear motion, and, usually, to transmit power. Familiar applications include the lead screws of lathes, and the screws for vises, presses, and jacks.

An application of power screws to a power-driven jack is shown in Fig. 8–4. You should be able to identify the worm, the worm gear, the screw, and the nut. Is the worm gear supported by one bearing or two?

In Fig. 8–5 a square-threaded power screw with single thread having a mean diameter  $d_m$ , a pitch  $p$ , a lead angle  $\lambda$ , and a helix angle  $\psi$  is loaded by the axial compressive force  $F$ . We wish to find an expression for the torque required raising this load, and another expression for the torque required to lower the load.

First, imagine that a single thread of the screw is unrolled or developed (Fig. 8–6) for exactly a single turn. Then one edge of the thread will form the hypotenuse of a right triangle whose base is the circumference of the mean-thread-diameter circle and whose height is the lead. The angle  $\lambda$ , in Figs. 8–5 and 8–6, is the lead angle of the thread. We represent the summation of all the axial forces acting upon the normal thread area by  $F$ . To raise the load, a force  $P_R$  acts to the right (Fig. 8–6a), and to lower the load,  $P_L$  acts to the left (Fig. 8–6b). The friction force is the product of the coefficient of friction  $f$  with the normal force  $N$ , and acts to oppose the motion. The system is in equilibrium under the action of these forces, and hence, for raising the load, we have

Finally, noting that the torque is the product of the force  $P$  and the mean radius  $d_m/2$ , for raising the load we can write

$$T_R = \frac{Fd_m}{2} \left( \frac{l + \pi f d_m}{\pi d_m - fl} \right) \quad (5-1)$$

Where  $T_R$  is the torque required for two purposes: to overcome thread friction and to raise the load.

The torque required to lower the load is found to be

$$T_L = \frac{Fd_m}{2} \left( \frac{\pi f d_m - l}{\pi d_m + fl} \right) \quad (5-2)$$

This is the torque required to overcome a part of the friction in lowering the load. It may turn out, in specific instances where the lead is large or the friction is low, that the load will lower itself by causing the screw to spin without any external effort.

In such cases, the torque  $T_L$  from Equation (5-2) will be negative or zero. When a positive torque is obtained from this equation, the screw is said to be self-locking. Thus the condition for **self-locking** is

$$\pi f d_m > l$$

Now divide both sides of this inequality by  $\pi d_m$ . We get

$$f > \tan \lambda$$

This relation states that self-locking is obtained whenever the coefficient of thread friction is equal to or greater than the tangent of the thread lead angle.

An expression for efficiency is also useful in the evaluation of power screws. If we let  $f = 0$  in Equation (5-1), we obtain

$$T_0 = \frac{Fl}{2\pi}$$

Which, since thread friction has been eliminated, is the torque required only to raise the load.

The **thread efficiency** is thus defined as

$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi T_R}$$

The preceding equations have been developed for square threads where the normal thread loads are parallel to the axis of the screw. In the case of Acme or other threads, the normal thread load is inclined to the axis because of the *thread angle*  $2\alpha$  and the *lead angle*  $\lambda$ .

Since lead angles are small, this inclination can be neglected and only the effect of the thread angle (Figure 8-7a) considered. The effect of the angle  $\alpha$  is to increase the frictional force by the wedging action of the threads. Therefore the frictional terms in Equation (5-1) must be divided by  $\cos \alpha$ . For raising the load, or for tightening a screw or bolt, this yields

$$T_R = \frac{Fd_m}{2} \left( \frac{l + \pi f d_m \sec \alpha}{\pi d_m - fl \sec \alpha} \right) \quad (5-3)$$

In using Equation (5-3), remember that it is an approximation because the effect of the lead angle has been neglected.

For power screws, the Acme thread is not as efficient as the square thread, because of the additional friction due to the wedging action, but it is often preferred because it is easier to machine and permits the use of a split nut, which can be adjusted to take up for wear.



Usually a third component of torque must be applied in power-screw applications. When the screw is loaded axially, a **thrust** or **collar** bearing must be employed between the rotating and stationary members in order to carry the axial component. Figure 8–7b shows a typical thrust collar in which the load is assumed to be concentrated at the *mean collar diameter*  $d_c$ . If  $f_c$  is the *coefficient of collar friction*, the torque required is

$$T_c = \frac{F f_c d_c}{2} \quad (5-4)$$

Nominal body stresses in power screws can be related to thread parameters as follows. The **maximum nominal shear stress**  $\tau$  in torsion of the **screw** body can be expressed as

$$\tau = \frac{16T}{\pi d_r^3}$$

The direct stresses **tensile or compressive axial stress**  $\sigma$  in the body of the **screw** due to load F is

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_r^2}$$

The **direct shear stresses**  $\tau$  in the body of the **screw** due to load F is

$$\tau = F/A_s$$

where  $A_s$  is the shear area =  $p \cdot (d_r/2)^2$

Nominal thread stresses in power screws can be related to thread parameters as follows. The **bearing stress (crushing stress)**,  $\sigma_B$  ( $\sigma_C$ ), is from the force F pressing into the surface area of the **thread**, as in Fig. 8–8, giving

$$\sigma_B = \frac{F}{\pi d_m n_t p / 2} = \frac{2F}{\pi d_m n_t p} \quad (5-5)$$

Where  $n_t$  is the number of engaged threads. The **bending stress** at the root of the **thread**  $\sigma_b$  (**screw**), in the  $x$  direction, is found from

$$\sigma_b = \frac{M}{Z} = \frac{Fp}{4} \frac{24}{\pi d_r n_t p^2} = \frac{6F}{\pi d_r n_t p} \quad (5-6)$$

The **transverse shear stress**  $\tau$  at the **center of the root of the thread** due to load F is

$$\tau = \frac{3V}{2A} = \frac{3}{2} \frac{F}{\pi d_r n_t p / 2} = \frac{3F}{\pi d_r n_t p} \quad (5-7)$$

and at the top of the root it is zero. The von Mises stress  $\sigma'$  at the top of the root “plane” is found by first identifying the orthogonal normal stresses and the shear stresses. From the coordinate system of Fig. 8–8, we note

$$\begin{aligned}\sigma_x &= \frac{6F}{\pi d_r n_t p} & \tau_{xy} &= 0 \\ \sigma_y &= 0 & \tau_{yz} &= \frac{16T}{\pi d_r^3} \\ \sigma_z &= -\frac{4F}{\pi d_r^2} & \tau_{zx} &= 0\end{aligned}\tag{5-8}$$

then use

$$\sigma' = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2}\tag{5-9}$$

Alternatively, you can determine the principal stresses and then use Eq. (5-9) to find the von Mises stress.

The screw-thread form is complicated from an analysis viewpoint. Remember the origin of the tensile-stress area  $A_t$ , which comes from experiment. A power screw lifting a load is in compression and its thread pitch is shortened by elastic deformation. Its engaging nut is in tension and its thread pitch is lengthened. The engaged threads cannot share the load equally. Some experiments show that the *first* engaged thread carries 0.38 of the load, the *second* 0.25, the *third* 0.18, and the seventh is free of load. In estimating thread stresses by the equations above, substituting 0.38F for F and setting  $n_t$  to 1 will give the largest level of stresses in the thread-nut combination.

Table 8-4 shows safe bearing pressures on threads, to protect the moving surfaces from abnormal wear. Table 8-5 shows the coefficients of sliding friction for common material pairs. Table 8-6 shows coefficients of starting and running friction for common material pairs.

### 5.3 Bolt Strength

In the specification standards for bolts, the strength is specified by stating SAE or ASTM minimum quantities, the minimum proof strength, or minimum proof load, and the minimum tensile strength. The proof load is the maximum load (force) that a bolt can withstand without acquiring a permanent set. The proof strength is the quotient of the proof load and the tensile stress area. Tables 8-9, 8-10, and 8-11 provide minimum strength specifications for steel bolts.

The SAE specifications are found in Table 8-9. The bolt grades are numbered according to the tensile strengths, with decimals used for variations at the same strength level. Bolts and screws are available in all grades listed. Studs are available in grades 1, 2, 4, 5, 8, and 8.1. Grade 8.1 is not listed.

ASTM specifications are listed in Table 8-10. ASTM threads are shorter because ASTM deals mostly with structures; structural connections are generally loaded in shear, and the decreased thread length provides more shank area.

Specifications for metric fasteners are given in Table 8-11.

#### 5.4 Bolted and Riveted Joints Loaded in Shear:

The riveted joints can be classified into *Lap joint* (two plates) and *Butt joints* (more than two plates). Riveted and bolted joints loaded in shear are treated exactly alike in design and analysis.

Figure 8–23a shows a riveted connection loaded in shear. Let us now study the various means by which this connection might fail.

Figure 8–23b shows a failure by *bending of the rivet* or of the riveted members. The bending moment is approximately  $M = F.t/2$ , where  $F$  is the shearing force and  $t$  is the grip of the rivet, that is, the total thickness of the connected parts. The bending stress in the members or in the rivet is, neglecting stress concentration,

$$\sigma = \frac{M}{I/c} \quad (5-8)$$

where  $I/c$  is the section modulus for the weakest member or for the rivet or rivets, depending upon which stress is to be found. The calculation of the bending stress in this manner is an assumption, because we do not know exactly how the load is distributed to the rivet or the relative deformations of the rivet and the members. Although this equation can be used to determine the bending stress, it is seldom used in design; instead its effect is compensated for by an increase in the factor of safety.

In Fig. 8–23c failure of the *rivet* by *pure shear* is shown; the stress in the rivet is

$$\tau = \frac{F}{A} \quad (5-9)$$

where  $A$  is the cross-sectional area of *all* the rivets in the group. It may be noted that it is standard practice in structural design to use the nominal diameter of the rivet rather than the diameter of the hole, even though a hot-driven rivet expands and nearly fills up the hole.

*Rupture* of one of the connected members or plates by pure *tension* is illustrated in Fig. 8–23d. The tensile stress is

$$\sigma = \frac{F}{A} \quad (5-10)$$

where  $A$  is the net area of the plate, that is, the area reduced by an amount equal to the area of all the rivet holes. The stress-concentration effects are not considered in structural design, because the loads are static and the materials ductile.

In calculating the area for Eq. (5–10), the designer should, of course, use the combination of rivet or bolt holes that give the smallest area.

Figure 8–23e illustrates a failure by *crushing* of the rivet or plate. Calculation of this stress, which is usually called a *bearing stress*, is complicated by the distribution of the load on the cylindrical surface of the rivet. The exact values of the forces acting upon the rivet are unknown, and so it is customary to assume that the components of these forces are uniformly distributed over the projected contact area of the rivet. This gives for the stress

$$\sigma = \frac{F}{A} \quad (5-11)$$

where the projected area for a single rivet is  $A = td$ . Here,  $t$  is the thickness of the thinnest plate and  $d$  is the rivet or bolt diameter.

Edge shearing, or tearing, of the margin is shown in Fig. 8–23f and g, respectively. In structural practice this failure is avoided by spacing the rivets at least 1.5 diameters away from the edge. Bolted connections usually are spaced an even greater distance than this for satisfactory appearance, and hence this type of failure may usually be neglected.

In a rivet joint, the rivets all share the load in shear, bearing in the rivet, bearing in the member, and shear in the rivet. Other failures are participated in by only some of the joint. In a bolted joint, shear is taken by clamping friction, and bearing does not exist. When bolt preload is lost, one bolt begins to carry the shear and bearing until yielding slowly brings other fasteners in to share the shear and bearing.

### 5.5 Shear Joints with Eccentric Loading:

In the previous example, the load distributed equally to the bolts since the load acted along a line of symmetry of the fasteners. The analysis of a shear joint undergoing eccentric loading requires locating the center of relative motion between the two members. In Fig. 8–26 let  $A_1$  to  $A_5$  be the respective cross-sectional areas of a group of five pins, or hot-driven rivets, or tight-fitting shoulder bolts. Under this assumption the rotational pivot point lies at the centroid of the cross-sectional area pattern of the pins, rivets, or bolts. Using statics, we learn that the centroid  $G$  is located by the coordinates  $\bar{x}$  and  $\bar{y}$ , where  $x_i$  and  $y_i$  are the distances to the  $i$ th area center:

$$\begin{aligned} \bar{x} &= \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i x_i}{\sum_1^n A_i} \\ \bar{y} &= \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + A_5y_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i y_i}{\sum_1^n A_i} \end{aligned} \quad (5-12)$$

In many instances the centroid can be located by symmetry.

An example of eccentric loading of fasteners is shown in Fig. 8–27. This is a portion of a machine frame containing a beam subjected to the action of a bending load. In this case, the beam is fastened to vertical members at the ends with specially prepared load-sharing bolts. You will recognize the schematic representation in Fig. 8–27b as a statically indeterminate beam with both ends fixed and with moment and shear reactions at each end.

For convenience, the centers of the bolts at the left end of the beam are drawn to a larger scale in Fig. 8–27c. Point O represents the centroid of the group, and it is assumed in this example that all the bolts are of the same diameter. Note that the forces that are shown in Fig. 8–27c are the resultant forces acting on the pins with a net force and moment equal and opposite to the reaction loads  $V_1$  and  $M_1$  acting at O. The total load taken by each bolt will be calculated in three steps.

In the first step the shear  $V_1$  is divided equally among the bolts so that each bolt takes  $F' = V_1/n$ , where  $n$  refers to the number of bolts in the group and the force  $F'$  is called the direct load, or primary shear.

It is noted that an equal distribution of the direct load to the bolts assumes an absolutely rigid member. The arrangement of the bolts or the shape and size of the members sometimes justifies the use of another assumption as to the division of the load. The direct loads  $F'_n$  are shown as vectors on the loading diagram (Fig. 8–27c).

The moment load, or secondary shear, is the additional load on each bolt due to the moment  $M_1$ . If  $r_A$ ,  $r_B$ ,  $r_C$ , etc., are the radial distances from the centroid to the center of each bolt, the moment and moment loads are related as follows:

$$M_1 = F''_A r_A + F''_B r_B + F''_C r_C + \dots \quad (a)$$

where the  $F''$  are the moment loads. The force taken by each bolt depends upon its radial distance from the centroid; that is, the bolt farthest from the centroid takes the greatest load, while the nearest bolt takes the smallest. This is because the moment loads are caused by bearing pressure of the bolt holes against the bolts as the members rotate slightly around the centroid with respect to each other, with the amount of displacement at each bolt proportional to its radius from the centroid. We can therefore write

$$\frac{F''_A}{r_A} = \frac{F''_B}{r_B} = \frac{F''_C}{r_C} \quad (b)$$

where again, the diameters of the bolts are assumed equal. If not, then one replaces  $F''$  in Eq. (b) with the shear stresses  $\tau'' = 4F''/\pi d^2$  for each bolt. Solving Eqs. (a) and (b) simultaneously, we obtain

$$F''_n = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + \dots} \quad (5-13)$$

where the subscript  $n$  refers to the particular bolt whose load is to be found. Each moment load is a force vector perpendicular to the radial line from the centroid to the bolt center.

In the third step the direct and moment loads are added vectorially to obtain the resultant load on each bolt. Since all the bolts or rivets are usually the same size, only that bolt having the maximum load need be considered. When the maximum load is found, the strength may be determined by using the various methods already described.

## CHAPTER SIX

### Welding, Bonding, and the Design of Permanent Joints

Form can more readily pursue function with the help of joining processes such as welding, brazing, soldering, cementing, and gluing—processes that are used extensively in manufacturing today. Whenever parts have to be assembled or fabricated, there is usually good cause for considering one of these processes in preliminary design work. Particularly when sections to be joined are thin, one of these methods may lead to significant savings. The elimination of individual fasteners, with their holes and assembly costs, is an important factor. Also, some of the methods allow rapid machine assembly, furthering their attractiveness.

Riveted permanent joints were common as the means of fastening rolled steel shapes to one another to form a permanent joint. The childhood fascination of seeing a cherry-red hot rivet thrown with tongs across a building skeleton to be unerringly caught by a person with a conical bucket, to be hammered pneumatically into its final shape, is all but gone. Two developments relegated riveting to lesser prominence. The first was the development of high-strength steel bolts whose preload could be controlled. The second was the improvement of welding, competing both in cost and in latitude of possible form.

#### **6.1 Welding Symbols:**

A weldment is fabricated by welding together a collection of metal shapes, cut to particular configurations. During welding, the several parts are held securely together, often by clamping or jiggling. The welds must be precisely specified on working drawings, and this is done by using the welding symbol, shown in Figure 9-1, as standardized by the American Welding Society (AWS). The arrow of this symbol points to the joint to be welded. The *arrow side* of a joint is the line, side, area, or near member to which the arrow points. The side opposite the arrow side is the other side.

Figures 9-3 to 9-6 illustrate the types of welds used most frequently by designers. For general machine elements most welds are fillet welds, though butt welds are used a great deal in designing pressure vessels. Of course, the parts to be joined must be arranged so that there is sufficient clearance for the welding operation. If unusual joints are required because of insufficient clearance or because of the section shape, the design may be a poor one and the designer should begin again and endeavor to synthesize another solution.

Since heat is used in the welding operation, there are metallurgical changes in the parent metal in the vicinity of the weld. Also, residual stresses may be introduced because of clamping or holding or, sometimes, because of the order of welding. Usually these residual stresses are not severe enough to cause concern; in some cases a light heat treatment after welding has been found helpful in relieving them. When the parts to be welded are thick, a preheating will also be of benefit. If the reliability of the component is to be quite high, a testing program should be established to learn what changes or additions to the operations are necessary to ensure the best quality.

**6.2 Butt (groove) and Fillet Welds:**

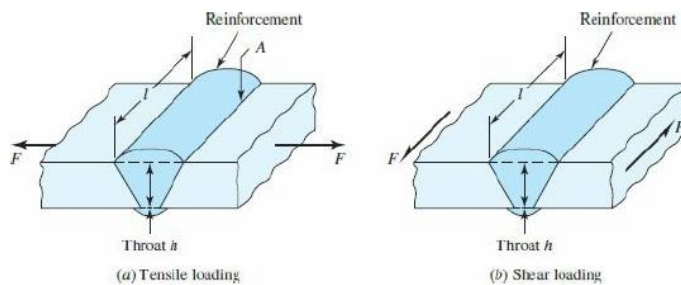


Figure (a) shows a single V-groove weld (Butt weld) loaded by the tensile force  $F$ . For either tension or compression loading, the average normal stress is

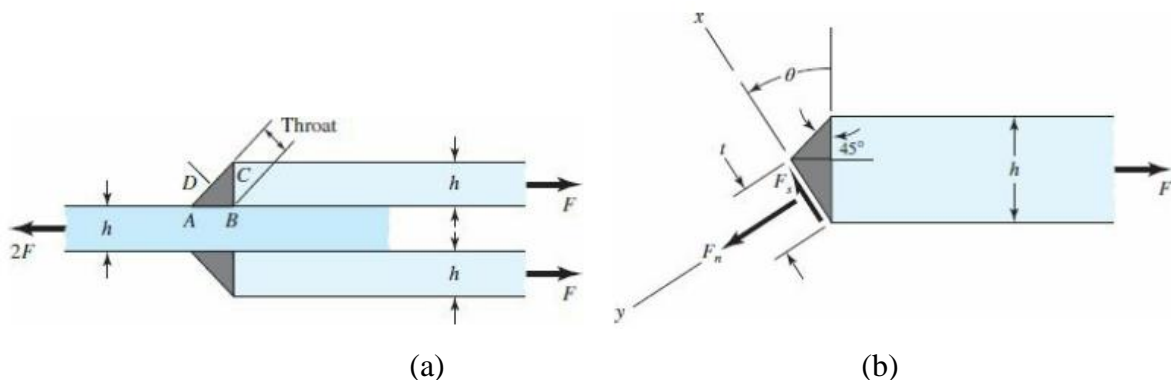
$$\sigma = \frac{F}{hl} \tag{6-1}$$

Where  $h$  is the *weld throat* and  $l$  is the *length of the weld*.

The average stress in a butt weld due to shear loading (Figure b) is

$$\tau = \frac{F}{hl} \tag{6-2}$$

Figure (a) below illustrates a typical transverse fillet weld. In Figure (b) a portion of the welded joint has been isolated from Figure (a) as a free body. At angle  $\theta$  the forces on each weldment consist of a normal force  $F_n$  and a shear force  $F_s$ .





Consider the external loading to be carried by shear forces on the throat area of the weld. By ignoring the normal stress on the throat, the shearing stresses are inflated sufficiently to render the model conservative.

$$\tau = \frac{F}{0.707hl} = \frac{1.414F}{hl}$$

6-3

which assumes the entire force  $F$  is accounted for by a shear stress in the minimum throat area.

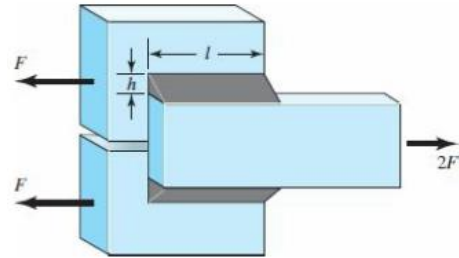
The throat area is

$$A = l \cdot \text{throat} (t)$$

$$= L \cdot h \cdot \cos 45^\circ$$

$$= 0.707 h \cdot l$$

Where  $h$  is the lead size.



### 5.3 Stresses in Welded Joints in Torsion:

Figure illustrates a cantilever welded to a column by two fillet welds each of length  $L$ . The reaction at the support of a cantilever always consists of a shear force  $V$  and a moment  $M$ .

The shear force produces a primary shear in the welds of magnitude

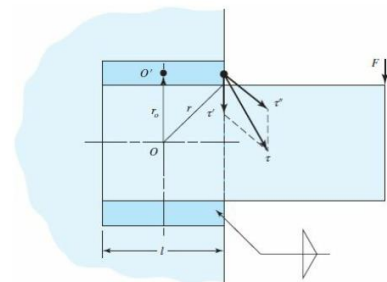
$$\tau' = \frac{V}{A}$$

where  $A$  is the *throat area of all the welds*.

The moment at the support produces secondary shear or torsion of the welds, and this stress is given by the equation

$$\tau'' = \frac{Mr}{J}$$

6-4



6-5

where  $r$  is the distance from the centroid of the weld group to the point in the weld of interest and  $J$  is the second polar moment of area of the weld group about the centroid of the group. When the sizes of the welds are known, these equations can be solved and the results combined to obtain the maximum shear stress. Note that  $r$  is usually the *farthest distance from the centroid of the weld group*.

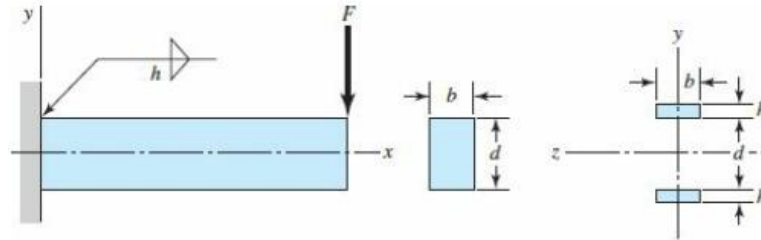
$$J = 0.707hJ_u$$

6-6

where  $J_u$  is a *unit second polar moment of area*. Table 9-1 lists the throat areas, location of centroid of the weld, and the unit second polar moments of area for the most common fillet welds encountered.

**5.4 Stresses in Welded Joints in Bending:**

Figure shows a cantilever welded to a support by fillet welds at top and bottom. A free body diagram of the beam would show a shear-force reaction  $V$  and a moment reaction  $M$ .



The shear force produces a primary shear in the welds of magnitude.

$$\tau' = \frac{V}{A}$$

6-7

where  $A$  is the total throat area.

The moment  $M$  induces a horizontal shear stress component in the welds. Treating the two welds of Figure as lines we find the unit second moment of area to be

$$I_u = \frac{bd^2}{2}$$

The second moment of area  $I$ , based on weld throat area, is

$$I = 0.707hI_u = 0.707h \frac{bd^2}{2}$$

The nominal throat shear stress is now found to be

$$\tau'' = \frac{Mc}{I} = \frac{Md/2}{0.707hbd^2/2} = \frac{1.414M}{bdh}$$

6-8

Table 9-2 lists the throat areas, location of centroid of the weld, and the unit second moments of area for the most common fillet welds encountered.

The vertical (primary) shear of Equation (6-7) and the horizontal (secondary) shear of Equation (6-8) are then combined as vectors to give

$$\tau = (\tau'^2 + \tau''^2)^{1/2}$$

6-9